Chapter 4

Exercises for Section 4.3

1. Spin the pointer a large number of times and count the proportion that fall in the grey sector. Yes, only if the spinner is balanced and the angle of the grey sector is known.

2. Yes, by tossing the thumb tack a large number of times and calculating the proportion landing point down. No, as the outcomes are not equally likely. We cannot argue from symmetry conditions as we could with the pointer.

3. **(a)** F (the number of 9s in a column will be random).

(b) T.

(c) F (previous outcomes do not affect the current outcome).

(d) F (all sequences are equally likely).

Exercises for Section 4.4.2

1. $A = \{(1,3), (2,2), (3,1)\}$.

2. $A = \{HHT, HHHHT, HHHHHHT, \ldots\}$. $\overline{A} = \{T, HT, HHHT, \ldots\}$,
$\overline{A} =$ "odd (or zero) number of tosses before the first tail."

3. $S = \{TT, THT, HTT, THHT, HTHT, THHH, HHH, HTHH, HHTT, HHTH\}$,
$A = \{THT, HTT, HHH\}$.

Exercises for Section 4.4.3

1. $S = \{(T1), (T2), \ldots, (T6), (H1), (H2), \ldots, (H6)\}$.
(i) $A = \{(T3), (H3)\}$. **(ii)** $B = \{(H1), (H2), (H3), (H4), (H5), (H6)\}$.
(iii) A and $B = \{(H3)\}$.
(iv) A or $B = \{(T3), (H1), (H2), (H3), (H4), (H5), (H6)\}$.

2. $S = \{(A,A), (A,B), (A,O), (A,AB), (B,A), (B,B), (B,O), (B,AB),$
$(O,A), (O,B), (O,O), (O,AB), (AB,A), (AB,B), (AB,O), (AB,AB)\}$.

(a) $C = \{(A,A), (B,B), (O,O), (AB,AB)\}$.

(b) $D = \{(A,A), (A,B), (A,O), (A,AB), (B,A), (O,A), (AB,A)\}$.

(c) C and $D = \{(A,A)\}$.

Exercises for Section 4.4.4

1. **(i)** Four of the 36 equally-likely outcomes add to 9 [namely (3,6), (4,5), (5,4) and (6,3)] so the probability is $\frac{4}{36} = \frac{1}{9}$.

(ii) Arguing similarly we get $\frac{18}{36} = \frac{1}{2}$.

2. **(a)** **(i)** $\frac{363}{5584} = 0.0650$. **(ii)** $\frac{911}{5584} = 0.1631$.

(b) $S = \{MW, MS, MP, FM, FS, FP\}$. The probability distribution is found by dividing each number in the table by 5,584, namely : 0.305, 0.214, 0.098, 0.217, 0.101, 0.065. (See Table 4.4.2.)

(c) $S = \{W, S, P\}$. The probability distribution is found by dividing the column sums by 5,584, namely : 0.522, 0.315, 0.163.

(d) It is easier to compute any probability that we might be interested in.

3. **(a)** People may lose more than one job.

(b) No, as there may be fewer females with jobs (which is likely).

(c) We need to compare the proportions of those with jobs losing their jobs for both males and females.

Exercises for Section 4.5.1

1. Let A = "wet" and B = "windy". We enter the numbers given to us into the following two-way table

	Wet (A)	Dry ($\overline{A}$)	Total
Windy (B)	0.2		0.4
Windless ($\overline{B}$)			
Total		0.7	1.0

and complete it to obtain

	Wet (A)	Dry ($\overline{A}$)	Total
Windy (B)	0.2	0.2	0.4
Windless ($\overline{B}$)	0.1	0.5	0.6
Total	0.3	0.7	1.0

Reading from the table, we get:

(a) $\text{pr}(A) = 1 - 0.7 = 0.3$.

(b) $\text{pr}(A \text{ or } B) = 0.2 + 0.2 + 0.1 = 0.5$.

(c) $\text{pr}(\overline{A} \text{ and } \overline{B}) = 0.5$. We can also get this from the complement of (b).

2. **(a)** 0.353 + 0.062 = 0.415.

(b) 0.195 + 0.016 = 0.211.

(c) Sum the entries in the first three rows and columns to get 0.895.

(d) 1 - 0.895 = 0.105.

(e) 0.111+0.195+0.008+0.021+0.022 +0.003+0.016= 0.376.

(f) 0.111 + 0.008 + 0.022 = 0.141.

3. We construct the following two-way table by entering the numbers given to us

	Too tall	Not too tall	Total
Attractive	0.18		
Not attractive			0.16
Total	0.24		1.00

and complete it to obtain

	Too tall	Not too tall	Total
Attractive	0.18	0.66	0.84
Not attractive	0.06	0.10	0.16
Total	(.24)	.76	1.00

Reading from the table, we get:

(a) 0.06.

(b) 0.66.

(c) 0.1.

4. We can use either Table 4.4.2 or, for more accuracy, Table 4.4.1 (as used below).

(a) $1 - \frac{1703}{5584} = 0.6950$.

(b) $\frac{1703+1196+548+1210}{5584} = 0.8340$.

(c) $\frac{1196+548}{5584} = 0.3123$.

Exercises for Section 4.6.1

1. **(a)** $\frac{19}{68} = 0.2794$.

(b) $\frac{19+33}{125} = \frac{52}{125} = 0.416$.

2. $\text{pr}(S \mid W) = \frac{\text{pr}(S \text{ and } W)}{\text{pr}(W)} = \frac{0.5}{0.7} = \frac{5}{7}$.

3. **(a)** **(i)** $\frac{564}{5584} = 0.1010$.

(ii) $\frac{564}{2137} = 0.2639$.

(b) **(i)** $\frac{1196}{5584} = 0.2142$.

(ii) $\frac{1196}{3447} = 0.3470$.

(iii) $\frac{1760}{5584} = 0.3152$.

(c) **(i)** $\frac{363}{2137} = 0.1699$.

(ii) $\frac{548}{3447} = 0.1590$.

(iii) $\frac{911}{5584} = 0.1631$.

4. **(a)** (i) 0.353. (ii) $\frac{0.195}{0.353} = 0.5524$. (iii) $\frac{0.195}{0.336} = 0.5804$. (iv) 0.195.

(b) **(i)** Divorced (from the female divorced column).

(ii) To get the conditional probabilities, we divide each entry in column two by the total 0.336. This does not affect the relative magnitudes of the entries.

(iii) Never married. $\frac{0.401}{0.554} = 0.7238$.

(c) **(i)** $\frac{0.401}{0.554} = 0.7238$.

(ii) 0.401.

(d) $\frac{0.195+0.024+0.008+0.016}{0.353+0.031} = 0.6328$.

5. $0.2267 + 0.0366 = 0.2633$.

Exercises for Section 4.6.2

1. Let H = "homeless" and S = "schizophrenic". Then $\text{pr}(H) = 0.008$, $\text{pr}(S) = 0.01$, and $\text{pr}(S \mid H) = \frac{1}{3}$.

2. $0.26 \times 0.77 = 0.2002$.

3. $\frac{2}{6} \times \frac{1}{5} = \frac{1}{15}$.

Exercises for Section 4.6.3

1. We construct the following two-way table

Race	Blood type AB	Not AB	Total
European	.0324×.85 (= .02754)		.85
Maori	.0177×.15 (= .002655)		.15
Total			1.00

and complete to produce

Race	Blood type AB	Not AB	Total
European	.027540	.822460	.85
Maori	.002655	.147345	.15
Total	.030195	.969805	1.00

From the table we obtain:
$\text{pr}(\textit{group AB}) = 0.0302$, or 3.02%.
$\text{pr}(\textit{maori} \mid \textit{group AB}) = \frac{0.002655}{0.030195} = 0.0879$, or 8.8%.

2. We construct the following two-way table

Child	Parents: Both	One only	Neither	Total
Left-handed	$\frac{1}{2} \times .02$	$\frac{1}{6} \times .2$	$\frac{1}{16} \times .78$	
Right-handed				
Total	.02	.20	.78	1

and complete to produce

Child	Parents: Both	One only	Neither	Total
Left-handed	$\frac{1}{2} \times .02$	$\frac{1}{6} \times .2$	$\frac{1}{16} \times .78$	.09208
Right-handed	$\frac{1}{2} \times .02$	$\frac{5}{6} \times .2$	$\frac{15}{16} \times .78$	.90792
Total	.02	.20	.78	1

From the table we obtain:
$\text{pr}(\textit{child is left-handed}) = 0.09208$.
$\text{pr}(\textit{neither parent left-handed} \mid \textit{child left-handed}) = \frac{0.78/16}{0.09208} = 0.5294$.

3. We construct the following two-way table

	Killed white	Killed black	Total
Death sentence	30		36
No death sentence	184		
Total			326

and complete to produce

	Killed white	Killed black	Total
Death sentence	30	6	36
No death sentence	184	106	290
Total	214	112	326

From the table we obtain:
The probability of getting the death sentence for killing a white is $\frac{30}{214}$ (= 0.1402). The probability for killing a black is $\frac{6}{112}$ (= 0.0536). The ratio is about 2.6.

Exercises for Section 4.7.1

1. $0.06 \times 0.14 = 0.0084$ (assuming independence). The events may not be independent. He may think he is safe, as she is on the pill!

2. As the two are chosen at random, we can construct the following table. The information given belongs in the row- and colum-total positions. Independence allows us to obtain the internal entries by multiplying corresponding row- and column-total entries.

	A	B	O	AB	Total
A	$.41 \times .41$	$.41 \times .11$	$.41 \times .45$	$.41 \times .03$	.41
B	$.11 \times .41$	$.11 \times .11$	$.11 \times .45$	$.11 \times .03$	.11
O	$.45 \times .41$	$.45 \times .11$	$.45 \times .45$	$.45 \times .03$	.45
AB	$.03 \times .41$	$.03 \times .11$	$.03 \times .45$	$.03 \times .03$	.03
Total	.41	.11	.45	.03	1.00

We obtain the answer by summing down the diagonal elements.
pr(*Same blood phenotype*) $= (0.41)^2 + (0.45)^2 + (0.11)^2 + (0.03)^2 = 0.3836$.

Exercises for Section 4.7.3

1. **(a)** $0.5 \times 0.75 \times 0.25 \times 0.70 = 0.065625$.

(b) It is hard to tell. Perhaps matching and curing may not be independent (a closer match may be more likely to cure). It depends on what is meant by "matching".

2. Let A = "sirens not working," B = "visual signals not noticed," C = "batteries run down," D ="power failure" and E ="routing switches shut off." If the sirens going off means that the visual warnings will be noticed (i.e., $\overline{A}$ implies $\overline{B}$), then A and B are not independent. Otherwise A and B are likely to be independent. We have the direct causal sequence $C \to D \to E$, so that any event that is independent of C is also independent of D and E. If the sirens go off, then C will not occur (i.e., $\overline{A}$ implies $\overline{C}$). The same is true if the visual warnings are noted. Thus A and B are indirectly related to C.

Review Exercises 4

1. **(a)** $S = \{\text{Yes}, \text{No}\}$.
(b) $S = \{0, 1, 2, \ldots\}$.
(c) $S = \{0, 1, 2, \text{unknown}\}$.
(d) $S = [a, b]$ depends on where you live, e.g., $[-20°C, +50°C]$.
(e) $S = \{0, 1, 2, \ldots\}$.
(f) $S = \{h_1, h_2, \ldots, h_{10}\}$.
(g) $S = \{$bus, car, train, boat, two-wheeler, walk, other$\}$. The "other" category would include combinations such as car, then bus.

2. The probabilities should add to 1. To do this the missing entry must be $1 - (.21 + .32 + .17 + .16 + .05) = 0.09$.

3. There are several ways of approaching this. One follows: $1 = \text{pr}(A) + \text{pr}(\overline{A}) = \frac{4}{3}\text{pr}(A)$, so that $\text{pr}(A) = \frac{3}{4}$.

4. **(a)** $E = \{1, 3, 5\}$.
(b) $\overline{D} = \{1, 4, 6\}$.
(c) A, B and D.
(d) B and D.
(e) **(i)** A or $C = \{1, 2, 3, 4\}$. **(ii)** C and $D = \{2, 3\}$. **(iii)** C and $\overline{D} = \{1\}$.

5. **(a)** $\frac{541}{856} = 0.6320$.
(b) $\frac{143}{856} = 0.1671$.
(c) $\frac{191}{288} = 0.6632$.
(d) $\frac{192}{284} = 0.6761$.
(e) $\frac{191}{541} = 0.3530$.
(f) The four respective proportions are (TW.sm) $\frac{86}{143} = 0.6014$; (TW.lg) $\frac{191}{288} = 0.6632$; (TS.sm) $\frac{72}{141} = 0.5106$; and (TS.lg) $\frac{192}{284} = 0.6761$.
We would prefer to use the typeset large (TS.lg) page format as it had the highest response rate.

6. **(a)** Chevalier de Méré's argument assumes that getting an ace on one die is mutually exclusive of getting an ace on another die. This is not true as it is possible for both events to happen together. In fact the events are independent.
(b) *Game 1*: Let A_i = Get an Ace on ith die.

$$\begin{aligned}
\text{pr}(\textit{Wins Game 1}) &= \text{pr}(\textit{Get at least 1 Ace}) \\
&= 1 - \text{pr}(\textit{Get no Aces}) \\
&= 1 - \text{pr}(\textit{Get no Aces on 1st die} \text{ and } \ldots \\
&\qquad \textit{Get no Aces on 4th die}) \\
&= 1 - \text{pr}(\overline{A}_1) \times \text{pr}(\overline{A}_2) \times \text{pr}(\overline{A}_3) \times \text{pr}(\overline{A}_4) \\
&= 1 - \left(\frac{5}{6}\right)^4 = 0.5177.
\end{aligned}$$

Game 2: Let A_i = Get a pair of Aces on ith roll of dice.

$$\begin{aligned}
\text{pr}(\textit{Wins Game 2}) &= \text{pr}(\textit{Get at least 1 pair of Aces}) \\
&= 1 - \text{pr}(\textit{Get no pairs of Aces}) \\
&= 1 - \text{pr}(\textit{Get no pair of Aces on 1st roll} \text{ and } \ldots \\
&\qquad \textit{Get no pair of Aces on 24th roll}) \\
&= 1 - \text{pr}(\overline{A}_1) \times \cdots \times \text{pr}(\overline{A}_{24}) \\
&= 1 - \left(\frac{35}{36}\right)^{24} = 0.4914.
\end{aligned}$$

7. (a) We construct the following 2-way table.

	Arts	Com	Sci	Eng	Law	Educ	Med
Female	.65×.3	.41×.19	.39 × .18	.15 × .07	.52 × .07	.82 × .06	.49 × .04
Male				.			
Total	.3	.19	.18	.07	.07	.06	.04

continued

Other	Total
.47 × .09	
.09	1.0000

and complete to form

	Arts	Com	Sci	Eng	Law	Educ	Med	Other	Total
Female	.195	.0779	.0702	.0105	.0364	.0492	.0196	.0423	.5011
Male	.105	.1121	.1098	.0595	.0336	.0108	.0204	.0477	.4989
Total	.3	.19	.18	.07	.07	.06	.04	.09	1.0000

From the table we obtain:

(b) 50.11%.

(c) **(i)** $\frac{0.195}{0.5011}$, or 38.91%. **(ii)** $\frac{0.0364}{0.5011}$, or 7.26%. **(iii)** $\frac{0.0105}{0.5011}$, or 2.10%.
(iv) $\frac{0.0492}{0.5011}$, or 9.82%.

8. Let A = "abused", D = "diagnosed as abused". We construct the following 2-way table

	A	Not A	Total
D	.9 × .01	.03 × .99	
Not D			
Total	0.01	0.99	1.0000

⟶

	A	Not A	Total
D	0.009	0.0297	0.0387
Not D	0.001	0.9603	0.9613
Total	0.01	0.99	1.0000

From the table we obtain:

(a) $\text{pr}(A \mid D) = \frac{0.009}{0.0387} = 0.2326$.

(b) No, as their rates of abuse will be different.

(c) Probably not as there is a greater awareness of the problem today.

9. 40% of 1% = 0.4 × 0.01 = 0.004 or 4 per thousand. Or more formally,

$$\begin{aligned}
&\text{pr}(\textit{Baby infec.} \text{ and } \textit{Mother HIV}) \\
&= \text{pr}(\textit{Baby infec.} \mid \textit{Mother HIV})\text{pr}(\textit{mother HIV}) \\
&= 0.4 \times 0.01 \quad \text{(i.e, four in every thousand births)}.
\end{aligned}$$

10. (a) $\frac{1}{10}$th of $\frac{1}{4} = \frac{1}{10} \times \frac{1}{4} = \frac{1}{40}$.

(b) Let S = "schizophrenic" and H = "homeless". We can construct the following two-way table.

	S	Not S	Total
H	$\frac{1}{3} \times .008$	$\frac{2}{3} \times .008$	
Not H			
Total	0.01	0.99	1.0000

$\longrightarrow$

	S	Not S	Total
H	0.002667	0.0053334	0.008
Not H	0.007333	0.984666	0.992
Total	0.01	0.99	1.0000

From the table we obtain $\text{pr}(H \mid S) = \frac{0.002667}{0.01} = 0.2667$, or nearly 27%.

(c) It suggests schizophrenia is mostly hereditary, but we cannot deny the possibility of enviromental effects.

(d) The proportion of parents who are both schizophrenic.

(e) (Difficult) We use 1% as the percentage of people with schizophrenia. Independence of those marrying as far as schizophrenia is concerned allows us to fill in the following table.

	Father		
Mother	S	Not S	Total
S			0.01
Not S			0.99
Total	0.01	0.99	1.0000

$\longrightarrow$

	Father		
Mother	S	Not S	Total
S	.0001	.0099	0.01
Not S	.0099	.9801	0.99
Total	0.01	0.99	1.0000

This tells us that for the parents, pr(*Both S*) = 0.0001, pr(*One S*) = 0.0198, and pr(*Neither S*) = 0.9801.

We now move on to construct a table of probabilities relating the parents and what happens to their children. There is one probability that we need to know but were not given, namely the probability that a child becomes schizophrenic if neither parent is schizophrenic. We will denote this probability by p.

	Parents			
Child	Both S	One S	Neither S	Total
S	$.4 \times .0001$	$.1 \times .0981$	$p \times .9801$	
Not S				
Total	0.001	0.0198	0.9801	1.0000

$$pr(\textit{both parents } S \mid \textit{child } S) = \frac{\text{pr}(\textit{child } S \text{ and } \textit{both } S)}{pr(\textit{child } S)} = \frac{0.4 \times 0.0001}{0.4 \times 0.0001 + 0.1 \times 0.0981 + p \times 0.9801}.$$

The answer depends upon the value of p.

(f) Not all people who eventually develop schizophrenia will have done so. All we can say is that it is less than 1%.

11. (a) To see how the course-work score related to the distribution of final grades.

(b) The set of all members of the class.

(c) 0.101. **(d)** $0.853 \times 0.101 = 0.08615$. **(e)** 0.853.

(f) $0.100 + 0.004 = 0.104$. **(g)** 0.339. **(h)** $0.339 \times 0.133 = 0.04509$. **(i)** 0.133.

(j) We use the following two-way table.

	Course-work mark						
	0–5	5^+–10	10^+–15	15^+–20	20^+–25	25^+–30	Total
Failed	0.912×0.027	0.797×0.046	0.539×0.181	0.133×0.339	0.003×0.306	0×0.101	
Passed							
Total	0.027	0.046	0.181	0.339	0.306	0.101	1.0000

continued

$$\text{pr}(\textit{Failed}) = 0.912 \times 0.027 + 0.797 \times 0.046 + 0.539 \times 0.181 + 0.133 \times 0.339 + 0.003 \times 0.306 + 0 \times 0.101 = 0.2049.$$

$\text{pr}(\textit{Passed}) = 1 - 0.2049 = 0.7951.$

(k) We use the following two-way table.

	Course-work mark						
	0–5	5^+–10	10^+–15	15^+–20	20^+–25	25^+–30	Total
A grade	0×0.027	0×0.046	0.004×0.181	0.067×0.339	0.268×0.306	0.853×0.101	
Not A							
Total	0.027	0.046	0.181	0.339	0.306	0.101	1.0000

continued

$$\text{pr}(\textit{A grade}) = 0 \times 0.027 + 0 \times 0.046 + 0.004 \times 0.181 + 0.067 \times 0.339 + 0.268 \times 0.306 + 0.853 \times 0.101 = 0.1916.$$

(l) $\text{pr}(25^+\text{–}30 | \textit{A grade}) = \dfrac{\text{pr}(\textit{A grade} \text{ and } 25^+\text{–}30)}{\text{pr}(\textit{A grade})} = \dfrac{0.853 \times 0.101}{0.1916} = 0.4497.$

12. **(a)** Breast.

(b) **(i)** Lung. **(ii)** Lung. **(iii)** Lung.

(c) $\frac{0.3}{46.3} = 0.00648.$

(d) **(i)** $\frac{93}{275} = 0.3382.$ **(ii)** $\frac{28.9}{275} = 0.1051.$

(e) **(i)** Prostate. **(ii)** Uterus.

(f) **(i)** Lung. $\frac{146}{168} = 0.8690.$ **(ii)** Lung, $\frac{93}{102} = 0.9118.$ **(iii)** Lung, $\frac{53}{66} = 0.8030.$

(g) Incidence and mortality rates remain constant from year to year. There is no change in the population age structure. These assumptions should be reasonable if the time periods involved are not very long and there are no factors acting which change incidence rates or major changes in treatment methods.

(h) The diseases with shorter survival periods are more serious.

13. We use the following two-way table

	Test Result		
	Positive	Negative	Total
Bowel cancer		$.4 \times .0005 = .0002$	.0005
No bowel cancer	$.4 \times .9995$		.9995
Total			1.0000

and complete it to produce

	Test Result		
	Positive	Negative	Total
Bowel cancer	$.0005 \times .6 = .0003$	$.0005 \times .4 = .0002$	.0005
No bowel cancer	$.9995 \times .4 = .3998$	$.9995 \times .6 = .5997$	.9995
Total	.4001	.5999	1.0000

(a) The proportion of bowel cancer cases overlooked is 0.40. (Information originally given.)

Now using the table, we obtain:

(b) pr(*Test positive*) = 0.4001.

(c) $\text{pr}(\textit{Bowel cancer} \mid \textit{Test positive}) = \frac{\text{pr}(\textit{Bowel cancer} \text{ and } \textit{Test positive})}{\text{pr}(\textit{Test positive})} = \frac{0.0003}{0.4001}$ $= 0.00075$.

(d) It is recommended that no animal meat be consumed for at least a day before the test is taken.

14. **(a)** The probability is $\frac{1}{200} \times 0.95 \times 0.6 = 0.00285$, i.e., 0.26%.

(b) We construct the following two-way table. Note that of the people with the gene marker, 60% of 95% get CaCo. Of those without the marker, 1 in 20 (or 5%) get CaCo.

	Gene marker	No gene marker	Total
CaCo	$(0.60 \times 0.95) \times 0.25$	0.05×0.75	
No CaCo			
Total	0.25	0.75	1.0000

From the table we obtain

$$\text{pr}(\textit{CaCo}) = (0.6 \times 0.95) \times 0.25 + 0.05 \times 0.75 = 0.180,$$

$$\begin{aligned} \text{pr}(\textit{Gene marker} \mid \textit{CaCo}) &= \frac{\text{pr}(\textit{Gene marker} \text{ and } \textit{CaCo})}{\text{pr}(\textit{CaCo})} \\ &= \frac{(0.6 \times 0.95) \times 0.25}{0.18} = \frac{1}{7}. \end{aligned}$$

(c) One quarter of 5 million people will consume a \$1,000 procedure, so the expected cost is $\frac{1}{4} \times 5\,\text{million} \times 1000 = \$1250\,\text{million}$.

15. **(a)** Let W and L denote a win and a loss respectively for player 1. The sample space is $S = \{WW, WLW, WLL, LWW, LWL, LL\}$.

(b) **(i)**
$$\begin{aligned} \text{pr}(A) &= \text{pr}(WW) + \text{pr}(WLW) + \text{pr}(LWW) \\ &= (.5)^2 + (.5)^3 + (.5)^3 = 0.25 + 0.125 + 0.125 = 0.5. \end{aligned}$$

(ii) $\text{pr}(B) = \text{pr}(WW) + \text{pr}(LL) = 0.25 + 0.25 = 0.5$.

(iii) $\text{pr}(A \text{ and } B) = \text{pr}(WW) = 0.25$.

(iv)
$$\begin{aligned} \text{pr}(A \text{ or } B) &= \text{pr}(WW) + \text{pr}(WLW) + \text{pr}(LWW) + \text{pr}(LL) \\ &= 0.25 + 0.125 + 0.125 + 0.25 = 0.75. \end{aligned}$$

(v) $\text{pr}(B \mid A) = \dfrac{\text{pr}(A \text{ and } B)}{\text{pr}(A)} = \dfrac{0.25}{0.5} = 0.5.$

(c) We have pr(A and B) = 0.25 and also $\text{pr}(A) \times \text{pr}(B) = 0.5 \times 0.5 = 0.25$, so A and B are independent.

(d) A and B are not mutually exclusive since they have an outcome, namely WW, in common.

16. Sample space $S = \{UUU, UUD, UDU, UDD, DUU, DUD, DDU, DDD\}$, where UUD means up on 1st day, up on 2nd day and down on 3rd day.

(a) $A = \{UUU, UUD, UDU, DUU\}$. Thus

$$\begin{aligned}
\text{pr}(A) &= \text{pr}(UUU \text{ or } UUD \text{ or } UDU \text{ or } DUU) \\
&= \text{pr}(UUU) + \text{pr}(UUD) + \text{pr}(UDU) + \text{pr}(DUU) \\
&= \text{pr}(U) \times \text{pr}(U) \times \text{pr}(U) + \text{pr}(U) \times \text{pr}(U) \times \text{pr}(D) \\
&\quad + \text{pr}(U) \times Pr(D) \times \text{pr}(U) + \text{pr}(D) \times \text{pr}(U) \times \text{pr}(U) \\
&\qquad \text{(since events are independent} \\
&= \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{1}{3} \times \frac{2}{3} + \frac{1}{3} \times \frac{2}{3} \times \frac{1}{3} + \frac{2}{3} \times \frac{1}{3} \times \frac{1}{3} \\
&= \frac{7}{27}.
\end{aligned}$$

(b) $B = \{DUU, DUD, DDU, DDD\}$ so that arguing as in (a), we find $\text{pr}(B) = \frac{18}{27} = \frac{2}{3}$.

(c) A and $B = \{DUU\}$, so pr(A and B) = $\text{pr}(DUU) = \frac{2}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{2}{27}$.

(d) $\text{pr}(A \mid B) = \dfrac{\text{pr}(A \text{ and } B)}{\text{pr}(A)} = \dfrac{2/27}{18/27} = \dfrac{2}{18} = \dfrac{1}{9}.$

(e) **(i)** We know pr(A and B) = $\frac{2}{27}$. However, $\text{pr}(A) \times \text{pr}(B) = \frac{7}{27} \times \frac{2}{3} = \frac{14}{81} \neq$ pr(A and B), so A and B are not independent.

(ii) A and B are not mutually exclusive as they have an outcome, namely DUU, in common.

17. **(a)** Let M_i = "Minor prize on the ith draw" and $\overline{M}$ be its complement (no minor prize on ith draw).

(i) $\text{pr}(\textit{No minor prizes}) = \text{pr}(\overline{M}_1 \overline{M}_2 \overline{M}_3)$
$= \text{pr}(\overline{M}_1 \text{ and } \overline{M}_2 \text{ and } \overline{M}_3)$ (translating notation)
$= \text{pr}(\overline{M}_1)\text{pr}(\overline{M}_2)\text{pr}(\overline{M}_3)$ (using independence)
$= 0.9^3 = 0.729.$

Remaining parts use the same types of argument as in (a)(i) but are given with less detail.

(ii) $\text{pr}(\textit{all 3 minor prizes}) = \text{pr}(M_1 M_2 M_3) = 0.1^3 = 0.001.$

(iii) $\text{pr}(\textit{exactly 1 minor prize}) = \text{pr}(M_1 \overline{M}_2 \overline{M}_3 \text{ or } \overline{M}_1 M_2 \overline{M}_3 \text{ or } \overline{M}_1 \overline{M}_2 M_3)$
$= \text{pr}(M_1 \overline{M}_2 \overline{M}_3) + \text{pr}(\overline{M}_1 M_2 \overline{M}_3) + \text{pr}(\overline{M}_1 \overline{M}_2 M_3)$ (as mutually exclusive)
$= 0.1 \times 0.9^2 + 0.9 \times 0.1 \times 0.9 + 0.9^2 \times 0.1$
$= 3 \times 0.1 \times 0.9^2 = 0.243.$

(iv) pr(*exactly 2 minor prizes*) = pr($M_1 M_2 \overline{M}_3$ or $M_1 \overline{M}_2 M_3$ or $M_1 M_2 \overline{M}_3$)
= pr($M_1 M_2 \overline{M}_3$) + pr($M_1 \overline{M}_2 M_3$) + pr($M_1 M_2 \overline{M}_3$) (as mutually exclusive)
$= 3 \times 0.1^2 \times 0.9 = 0.027$.

(b) Let L = "wins Las Vegas" and H = "wins Hawaii".

(i) pr($\overline{L}$ and $\overline{H}$) $= \frac{349}{350} \times \frac{99}{100} = 0.9872$.

(ii) pr(L and $\overline{H}$) $= \frac{1}{100} \times \frac{349}{350} = 0.009971$.

(iii) pr(L and H) $= \frac{1}{350} \times \frac{1}{100} = 2.857 \times 10^{-5}$.

(c) The process of picking major prizes is independent of that for picking minor prizes.

(i) pr(*No minor prizes* and *No major prizes*)
= pr(*No minor prizes*) × pr(*No major prizes*) = 0.729 × 0.9872 = 0.7197.
We have used independence and the answers to (a)(i) and (b)(i).

(ii) pr(*1 or more minor prizes* and *No major prizes*)
= pr(*1 or more minor prizes*) × pr(*No major prizes*) = (1 − 0.729) × 0.9872
= 0.2675.
(Note that 1 or more minor prizes is the complement of no minor prizes.)

18. Blast radius r = 45.5 metres, so blast area $= \pi r^2 = \pi \times 45.5^2 \approx 6504$ m^2.
Target area = 80000 × 48000 = 3840 million m^2.

(a) pr(*Being hit by 1 missile*) $= \frac{6504}{3{,}840{,}000{,}000} = 1.694 \times 10^{-6}$.

(b) pr(*All missiles miss*) = pr(*1st misses* and *2nd misses* and ... and *20th misses*)
= pr(*1st misses*) × pr(*2nd misses*) × ... pr(*20th misses*) (using independence)
$= (0.9999983)^{20} = 0.999966$.

19. **(a)** We form the following two-way table

	Low	Medium	High	Total
Claim	0.01 × 0.2	0.04 × 0.7	0.1 × 0.1	
No claim				
Total	0.2	0.7	0.1	1.00

and complete it to form

	Low	Medium	High	Total
Claim	0.002	0.028	0.01	0.04
No claim	0.198	0.672	0.09	0.96
Total	0.200	0.700	0.10	1.00

(b) pr(*Claim* | *Medium*) = 0.04 (information originally given).

(c) pr(*Claim* and *Medium*) = 0.028.

(d) pr(*Claim*) = 0.04.

(e) pr(*High* | *Claim*) $= \dfrac{\text{pr}(\textit{High} \text{ and } \textit{Claim})}{\text{pr}(\textit{Claim})} = \dfrac{0.01}{0.04} = 0.25$.

***(f)** Arguing as in problem 17(a)(i), the probability a low risk person has no claims in 3 years is 0.99^3. Similarly, for medium risk people it is 0.96^3 and for high risk people 0.9^3. We use this to create the following two-way table.

	Low	Medium	High	Total
No claims in 3 years	$0.99^3 \times 0.2$	$0.96^3 \times 0.7$	$0.9^3 \times 0.1$	
Some claims				
Total	0.2	0.7	0.1	1.00

From the table we see that
$\text{pr}(\textit{No claim in 3 years}) = 0.99^3 \times 0.2 + 0.96^3 \times 0.7 + 0.9^3 \times 0.1$
and thus $\text{pr}(\textit{Low} \mid \textit{No claims in 3 years}) = \dfrac{\text{pr}(\textit{Low} \text{ and } \textit{No claims in 3 years})}{\text{pr}(\textit{No claim in 3 years})}$

$$= \frac{0.99^3 \times 0.2}{0.99^3 \times 0.2 + 0.96^3 \times 0.7 + 0.9^3 \times 0.1} = 0.2190.$$

20. Individual answers. In (c) you muolitply the probabilities together.

21. **(a)** **(i)** $\frac{97,473}{98,826} = 0.9863.$

(ii) $\frac{89,099}{98,304} = 0.9064.$

(b) **(i)** 20 to 25 where the probability of dying is $\frac{97473-96594}{97473} = 0.0090.$

(ii) 35 to 40 where the probability of dying is $\frac{97317-96792}{97317} = 0.0054.$
Young men are risk takers.

(c) **(i)** $\frac{82690}{97473} \times \frac{89099}{98304} = 0.84834 \times 0.90636 = 0.7689.$

(ii) $\frac{97473-82690}{97473} \times \frac{89099}{98304} = 0.15166 \times 0.90636 = 0.1375.$

(iii) $\frac{97473-82690}{97473} \times \frac{98304-89099}{98304} = 0.15166 \times 0.09364 = 0.0142.$

(d) We have assumed that the life times of the man and the woman are independent. No. Positive. Keep each other going!? Two people in a couple will tend to be more similar in diet and lifestyle factors than two random people and they will have more similar exposures to hazards.

(e) A proportion $\frac{96792}{98304}$ of males who are alive at 20 live to 40. The corresponding proportion for females is $\frac{96792}{98304}$. We construct the following two-way table that applies only to people still alive at 20.

	Male	Female	Total
Live till 40	$\frac{94407}{97473} \times 0.5$	$\frac{96792}{98304} \times 0.5$	
Die before 40			
Total	0.5	0.5	1.00

From this we obtain $\text{pr}(\textit{Live till 40}) = \frac{94407}{97473} \times 0.5 + \frac{96792}{98304} \times 0.5 = 0.9766.$

(f) We assume that the survival rates haven't changed since the data were compiled. However, we can expect life expectancies, and hence the survival rates, to have increased a little. Still, the answers will give a reasonable approximation. The realities are rather subtle. Life-table data reflects current death rates for people at each age at the time the data was collected. For example, the people now dying or not dying at age 50 were 20-years old 30 years ago. The environment (including diet, disease patterns and many other things) they have lived through may be quite different from the environment current 20-year olds will experience over the next 30 years and such differences may produce different death rates for 50 year-olds. So these predictions depend on death rates for people of every age being the same as they were in 1985–7.

22. As the husband and wife each follow the same phenotype distribution, their blood types are independent.

(a) 0.09, as the wife's blood type does not affect (or is independent of) the husband's blood type.

We construct the following two-way table to help us with the remaining parts of the problem. The information we were given about blood group distributions belongs in the row and column totals of the table. Independence of husband and wife's blood types then allows us to get the central entries of the table by multiplying (cf. Example 4.7.3).

		Wife				
		A	B	O	AB	Total
Husband	A	0.1600	0.0360	0.1960	0.0080	0.40
	B	0.0360	0.0081	0.0441	0.0018	0.09
	C	0.1960	0.0441	0.2401	0.0098	0.49
	AB	0.0080	0.0018	0.0098	0.0004	0.02
	Total	0.40	0.09	0.49	0.02	1.00

(b) 0.0081 (reading from the table).

(c) Using the table, this is the sum all the probabilities in the second row and column which is the same as $0.09 + 0.09 - 0.0081 = 0.1719$.
Without using the table, the probability is 1 - pr(*Both do not have type B blood*) $= 1 - 0.91^2 = 0.1719$.

(d) Sum of the diagonal elements is $0.16 + 0.0081 + 0.2401 + 0.0004$, or 0.4086.

(e) The probability the partner is B or O which is $(0.09 + 0.49) = 0.58$. (We do not need the table for this.)

23. Suppose door 1 is the car (C) and doors 2 and 3 are goats (G). If you never switch, $\text{pr}(C) = \frac{1}{3}$. Now,

$$\begin{aligned}\text{pr}(C) &= \text{pr}(C \mid \textit{Choose 2 or 3})\text{pr}(\textit{Choose 2 or 3}) \\ &\quad +\text{pr}(C \mid \textit{Choose 1})\text{pr}(\textit{Choose 1}).\end{aligned}$$

If you always switch, then initially choosing 2 or 3 will guarantee that you get the car when you switch (you will have initially chosen a goat and the host will have chosen the other goat), i.e., $\text{pr}(C \mid \textit{Choose 2 or 3}) = 1$. However, if you initially chose 1 then you won't get the car when you switch, i.e., $\text{pr}(C \mid \textit{Choose 1}) = 0$. Hence

$$\text{pr}(C) = 1 \times \frac{2}{3} + 0 \times \frac{1}{3} = \frac{2}{3}.$$

24. Let drawer 1 (D1) contain two gold coins, drawer 2 (D2) contain two silver coins, and drawer 3 (D3) contain one gold and one silver coin. We construct the following two-way table

	Result		
Drawer	Gold	Silver	Total
2 gold	$1 \times \frac{1}{3}$		$\frac{1}{3}$
1 gold, 1 silver	$\frac{1}{2} \times \frac{1}{3}$		$\frac{1}{3}$
2 silver	$0 \times \frac{1}{3}$		$\frac{1}{3}$
Total			1

and complete it to obtain

	Result		
Drawer	Gold	Silver	Total
2 gold	$\frac{1}{3}$	0	$\frac{1}{3}$
1 gold, 1 silver	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$
2 silver	0	$\frac{1}{3}$	$\frac{1}{3}$
Total	$\frac{1}{2}$	$\frac{1}{2}$	1

$$\text{pr}(\textit{2-gold-coin drawer} \mid \textit{coin is gold}) = \frac{\text{pr}(\textit{2-gold-coin drawer} \text{ and } \textit{coin is gold})}{\text{pr}(\textit{coin is gold})}$$
$$= \frac{1}{3}/\frac{1}{2} = \frac{2}{3}.$$

25. (a) We construct the following two-way table

	Result		
Digit	Correct	Not corect	Total
0		0.001×0.4	0.4
1		0.002×0.6	0.6
Total			1.0

and complete it to obtain

	Correct	Not corect	Total
0	$0.999 \times 0.4 = 0.3996$	$0.001 \times 0.4 = 0.0004$	0.4
1	$0.998 \times 0.6 = 0.5988$	$0.002 \times 0.6 = 0.0012$	0.6
Total	0.9984	0.0016	1.0

Reading from the table we obtain pr(*Correct*) = 0.9984.

(b) **(i)** $0.9984^{1000} = 0.20164$. **(ii)** $0.9984^{10000} = 1.11 \times 10^{-7}$.

(c) The probability of reversing twice will be negligible so that we can ignore this event. The required probability is then $0.4 \times 0.999 \times 0.998 + 0.6 \times 0.998 \times 0.996 = 0.3988 + 0.5964 = 0.9952$.

26. (a) The system fails only if all components fail. The probability is $0.01 \times 0.02 \times 0.08 = 0.000016$.

(b) Power disconnection.

(c) Lighting systems. Some Christmas tree lights are in parallel while others may be in series (see below) so that when one goes out they all go out.

(d)
$$\begin{aligned}\text{pr}(\textit{System fails}) &= \text{pr}(\textit{At least one fails}) \\ &= 1 - \text{pr}(\textit{None fail}) \\ &= 1 - 0.98 \times 0.95 \times 0.92 \times 0.97 = 0.1692.\end{aligned}$$

[Note that the reliability of a system is the probability it works (complement of fails). The reliability of the system in (a), the probability the system is $1 - 0.000016 = 0.999984$, which is much higher than the reliability of any individual component. (The most reliable component is the light, which has reliability $1 - 0.01 = 0.99$.) The reliability of the system in (b) is $1 - 0.1692 = 0.8308$, which is much less reliable than the most reliable component. (The most reliable component is the inital drive, which has reliability $1 - 0.02 = 0.98$.)

(e) See (c).

***27.** Let pr(*Accident* | *Right handed*) $= p$. Then, pr(*Accident* | *Left handed*) $= 1.89p$. We put this information on a two-way table as follows

	Accident	No accident	Total
Left handed	$1.89p \times 0.1$		0.1
Right handed	$p \times 0.9$		0.9
Total			1.0

and partially complete it to form

	Accident	No accident	Total
Left handed	$1.89p \times .1$	*	0.1
Right handed	$p \times .9$	*	0.9
Total	$p \times 1.089$	*	1.0

(We don't need to know the items labeled * as they do not come into our calculations.) Then

$$\begin{aligned}\text{pr}(\textit{Left handed} \mid \textit{Accident}) &= \frac{\text{pr}(\textit{Left handed} \text{ and } \textit{Accident})}{\text{pr}(\textit{Accident})} \\ &= \frac{1.89p \times 0.1}{1.089p} = \frac{0.189}{1.089} = 0.1736.\end{aligned}$$

Note that the answer does not involve p.