CROWN and ANCHOR

Pay $10 to play each game. In one game, how much can I **expect** to win?

I.e., if I played the game a very large number of times, how much would I win per game **on average**?

Rules of the Game:

1. Pick any face of the die: heart; club; spade; diamond; crown; anchor.

   Let's say I pick hearts.

2. Toss the die three times:

   If no hearts come up I lose my money. If 1 heart comes up I get $10 plus my $10.

   If 2 hearts come up I get $20 plus my $10. If 3 hearts come up I get $30 plus my $10.

Let H_i be the event of throwing a heart on the *i*th throw.

My chances of gaining $30 in any ONE game

= my chances of throwing 3 hearts

= pr(H_1 and H_2 and H_3)

= pr(H_1)*pr(H_2)*pr(H_3) (by

= 1/6 * 1/6 * 1/6 indep.)

= 1/216

My chances of **losing** $10 in any ONE game

= my chances of throwing no hearts

= pr($\overline{H}_1$ and $\overline{H}_2$ and $\overline{H}_3$)

= pr($\overline{H}_1$)*pr($\overline{H}_2$)*pr($\overline{H}_3$)

= 5/6 * 5/6 * 5/6

= 125/216

My chances of gaining $20 in any ONE game

= my chances of throwing 2 hearts

$= \text{pr}\left(H_1 \text{ and } H_2 \text{ and } \overline{H}_3\right) + \text{pr}\left(H_1 \text{ and } \overline{H}_2 \text{ and } H_3\right) + \text{pr}\left(\overline{H}_1 \text{ and } H_2 \text{ and } H_3\right)$

$= \text{pr}(H_1)\times\text{pr}(H_2)\times\text{pr}(\overline{H}_3) + \text{pr}(H_1)\times\text{pr}(\overline{H}_2)\times\text{pr}(H_3) + \text{pr}(\overline{H}_1)\times\text{pr}(H_2)\times\text{pr}(H_3)$

(since the H_i are independent events)

$= 3\times\frac{1}{6}\times\frac{1}{6}\times\frac{5}{6}$

$= \frac{15}{216}$

My chances of gaining $10 in any ONE game

= my chances of throwing 1 heart

$= \text{pr}\left(H_1 \text{ and } \overline{H}_2 \text{ and } \overline{H}_3\right) + \text{pr}\left(\overline{H}_1 \text{ and } \overline{H}_2 \text{ and } H_3\right) + \text{pr}\left(\overline{H}_1 \text{ and } H_2 \text{ and } \overline{H}_3\right)$

$= \text{pr}(H_1)\times\text{pr}(\overline{H}_2)\times\text{pr}(\overline{H}_3) + \text{pr}(\overline{H}_1)\times\text{pr}(\overline{H}_2)\times\text{pr}(H_3) + \text{pr}(\overline{H}_1)\times\text{pr}(H_2)\times\text{pr}(\overline{H}_3)$

(since the H_i are independent events)

$= 3\times\frac{1}{6}\times\frac{5}{6}\times\frac{5}{6}$

$= \frac{75}{216}$

If I play **10000** games then I will gain:

$30 for about **1/216** of the games; -$10 for about **125/216** of the games;

$20 for about 15/216 of the games; $10 for about 75/216 of the games.

My gain in **10000** games

≅ $30*1/216*10000+$20*15/216*10000+$10*75/216+$-10*125/216*10000

= ($30*1/216+$20*15/216+$10*75/216+$-10*125/216)*10000

Hence my **expected** gain **per game**

= $30*1/216+$20*15/216+$10*75/216+$-10*125/216

= -$0.787

I.e. I can **expect** to **lose** about **79c** per game,

or put another way, over a very large number of games, I will **lose** about **79c** per game **on average**

In no actual game will I lose 79 cents. Because the probabilities are *long run* chances, the expected gain is also a *long run* figure.

Let ***X*** stand for my **gain in <u>any</u> one game**. ***X*** is called a **RANDOM VARIABLE.**
(It measures something random)

In this example, *X* can take on any one of the values: **$30, $20, $10, -$10**

x	30	20	10	-10
pr($X=x$)	1/216	15/216	75/216	125/216

This is the **probability function for *X*.**

Expected gain per game = E[gain per game] = E[*X*] = Expected value of *X*

What is the expected value of *X*?

$$E[X] = \$30\times\frac{1}{216} + \$20\times\frac{15}{216} + \$10\times\frac{75}{216} + -\$10\times\frac{125}{216}$$

= **-$0.79**

ie add up (each value * probability of that value occurring)

= $\sum x * \text{pr}(X=x)$

Discrete Random Variables

Chapter 5

RANDOM VARIABLES (r.v.'s) §5.1 page 197

A random variable, X, is a type of measurement taken on the outcome of an experiment.

PROBABILITY FUNCTIONS §5.2.1 page 198

The probability function for a discrete random variable X gives a probability, $\text{pr}(X = x)$, for each value x that X can take.

Note: $\text{pr}(X = x)$ is often written as **pr(*x*)**

MEAN OF A DISCRETE RANDOM VARIABLE *X*

Suppose X is a discrete random variable. The mean of X, or the expected value of X, is given by:

$$\mu_X = E(X)$$

$$= \sum x_i \, \text{pr}(X=x_i)$$

(formula sheet)

[μ said "mu"]

Example: Household Size 1996 Census

The following frequency table gives the number of occupants of private dwellings in the 1996 census.

No. of Occupants	Frequency	Percentage
0	8 235	0.6
1	264 363	20.7
2	420 027	32.9
3	215 670	16.9
4	201 951	15.8
5	102 408	8.0
6	38 892	3.0
7	13 776	1.1
8	11 010	0.9
Total	1 276 332	100

To find the mean number of people in a household based on the corresponding **frequency distribution** we use our calculator in statistics mode.

μ = mean = **2.73**

The standard deviation is given by

σ = **1.51**

[σ said "sigma"]

Using the **probability function**:

Let X be the number of people in a private dwelling

x	0	1	2	3	4	5	6	7	8
$\text{pr}(X = x)$	0.006	0.207	0.329	0.169	0.158	0.080	0.030	0.011	0.009

For the probability distribution of household size the mean is given by:

$\mu = E(X) = 0 * 0.006 + 1 * 0.207 + 2 * 0.329 + 3 * 0.169 + 4 * 0.158 + 5 * 0.080 + 6 * 0.030 + 7 * 0.011 + 8 * 0.009 = 2.73$

ie If we were to randomly select private dwellings and count the number of occupants, the (long term) average would be **2.73**

Note:

- The sum of the $\text{pr}(x)$'s over all values of x is **1**
- Probabilities, $\text{pr}(X = x)$, must have values between **0 and 1**
- μ_X is usually called the **population** mean.
- μ_X is the point where the bar graph of $\text{pr}(X = x)$ balances. See: Figure 5.4.2 page 218.

POPULATION STANDARD DEVIATION §5.4.2 page 219

The standard deviation of a discrete random variable X is given by:

$$\sigma_X = \text{sd}(X) = \sqrt{E[(X-\mu)^2]}$$

$$\sqrt{\sum (x_i-\mu)^2 * \text{pr}(X = x_i)}$$ **(formula sheet)**

σ_X measures the amount of spread/variation in the distribution of X about μ_X .

Example continued:

X has the same probability function as shown in the table above. What is the standard deviation of X?

$$\sigma_x = \text{sd}(X) = \sqrt{(0 - 2.73)^2*0.006 + (1 - 2.73)^2*0.207 + \ldots + (8 - 2.73)^2*0.009}$$

$$= \sqrt{2.2903} = 1.51$$

Example: Let X be the number of heads when two coins are tossed.

		1st Coin H	T	
		(.5*.5) (by independ.)		
2nd	H	0.25	0.25	0.5
	T	0.25	0.25	0.5
		0.5	0.5	1

x	0	1	2
pr(X=x)	0.25	0.5	0.25

$\mu_X = E(X) =$

0*025 + 1*0.5 + 2*0.25 = 1

$\sigma_X = sd(X) =$

$\sqrt{(0-1)^2 * 0.25 + (1-1)^2 * 0.5 + (2-1)^2 * 0.25} = \sqrt{0.5} = 0.707$

SKILLS IN MANIPULATING PROBABILITIES §5.2.5 page 200

Example: Below is the probability function for the number of people living in a private dwelling from the 1996 census.

Let X be the number of people in a private dwelling

x	0	1	2	3	4	5	6	7	8
pr(X=x)	0.006	0.207	0.329	0.169	0.158	0.080	0.030	0.011	0.009

Find the probability that the number of people living in a private dwelling was:

(a) more than 6 **pr(X>6) = 0.011 + 0.009 = 0.02**

(b) no more than 4 **pr(X<=4) = .006 + .207 + .329 +.169 + .158=.869**

(c) at least 2 **pr(X>=2) = 1 – 0.006 – 0.207 = 0.787**

(d) more than 3 but no more than 6 **pr(3<X<=6) = .158 + .080 + .030 = .268**

(e) between 6 and 10 (inclusive). **pr(6<=X<=*10*) = .030 + .011 + .009 = .05**

See: Example 5.2.5 page 200

Try: Exercises on §5.2 page 203

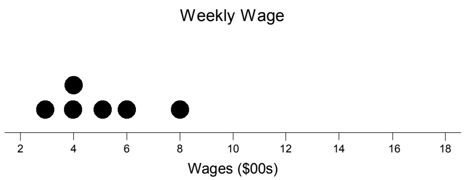
Read: Taking care with language page 201

MEAN AND STANDARD DEVIATION OF $Y = aX + b$ §5.4.3 page 220

Example

The weekly wages for a sample of workers in a factory, (in hundreds of dollars) are 5, 8, 4, 4, 3, 6.

Let $x_1 = 5$, $x_2 = 8$, $x_3 = 4$, $x_4 = 4$, $x_5 = 3$, $x_6 = 6$. A dot plot of these wages is:



"Centre" $\bar{x} = 5$

"Spread" $s_x \approx 1.79$

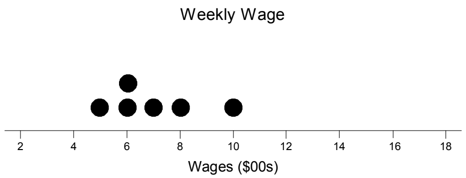
If each worker was given $200 more per week, what would happen to the mean and standard deviation?

New weekly wage = original weekly wage + 2

Let y_i be the new weekly wage.

$y_i = x_i + 2$

The weekly wages are now: 7, 10, 6, 6, 5, 8



"Centre" $\bar{y} = 7$

Ie, $\bar{y} = 5 + 2$

$\bar{y} = \bar{x} + 2$

"Spread" $s_y \approx 1.79$

Ie, $s_y = s_x$

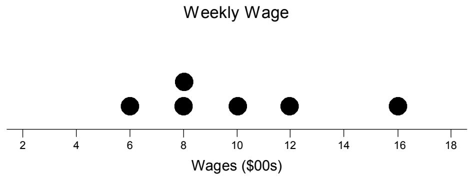
If each worker's weekly wage was doubled, what would happen to the mean and standard deviation?

New weekly wage $= 2 \times$ original weekly wage

Let y_i be the new weekly wage.

$y_i = 2x_i$

The weekly wages are now: 10, 16, 8, 8, 6, 12



"Centre" $\bar{y} = 10$

Ie, $\bar{y} = 2 \times 5$

$\bar{y} = 2\bar{x}$

"Spread" $s_y \approx 3.58$

Ie, $s_y \approx 2 \times 1.79$

$s_y = 2s_x$

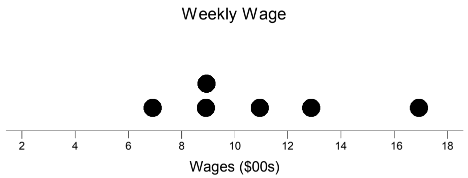
If the manager gave a Christmas bonus of double the worker's weekly wage plus $100 for extra spending, what would happen to the mean and standard deviation?

Bonus $= 2 \times$ original weekly wage + 1

Let y_i be the bonus.

$y_i = 2x_i + 1$

The bonuses are: 11, 17, 9, 9, 7, 13



"Centre" $\bar{y} = 11$

Ie, $\bar{y} = 2 \times 5 + 1$

$\bar{y} = 2\bar{x} + 1$

"Spread" $s_y \approx 3.58$

Ie, $s_y \approx 2 \times 1.79$

$s_y = 2s_x$

In general if $y_i = ax_i + b$, then

Centre" $\bar{y} = a\bar{x} + b$

Spread" $s_y = |a| s_x$

Means and standard deviations of probability distributions behave in exactly the same way.

$Y = aX + b$

$E(Y) = E(aX + b) = aE(X) + b$

$sd(Y) = sd(aX + b) = |a| sd(X)$

Example: Suppose X is a discrete random variable with $E(X) = 11$ and $sd(X) = 4$. Find $E(Y)$ and $sd(Y)$.

(a) $Y = 5X - 2$

$E(Y) = E(5X - 2)$
$= 5E(X) - 2$
$= 5 \times 11 - 2$
$= 53$

$sd(Y) = sd(5X - 2)$
$= |5| \times sd(X)$
$= 5 \times 4$
$= 20$

(b) $Y = 4 - 3X$

$E(Y) = E(4 - 3X)$
$= 4 - 3E(X)$
$= 4 - 3 \times 11$
$= -29$

$sd(Y) = sd(4 - 3X)$
$= |-3| \times sd(X)$
$= 3 \times 4$
$= 12$

See: Example 5.4.6 page 221

Try: Exercises on §5.4.3 page 221

Binomial Distribution **§5.3 pAGES 204 - 212**

The Frivolous Pursuits Quiz

True or false?

	Answer
1 Exposed to a wind of 30 miles per hour in a temperature of -30^o F, human flesh freezes solid in 30 seconds.	1 true
2 A coho is a fish	2 true
3 Before anaesthetics were invented, the shortest time recorded for a leg amputation was 20 sec by Napoleon's chief surgeon, Dominic Larrey. (13-15 secs)	3 false
4 A brool is a deep murmur	4 true

We assume that you guess each answer. The probability for getting a correct answer for each question is **0.5**

We wish to find a probability model for this type of situation.

Let X be the number of correctly guessed answers.

Number of correctly guessed answers, X	Frequency	Observed probability	Theoretical probability
0			1/16 = 0.0625
1			4/16 = 0.25
2			6/16 = 0.375
3			4/16 = 0.25
4			1/16 = 0.0625
Total			1

x	0	1	2	3	4
Possible outcomes	I I I I	C I I I I C I I I I C I I I I C	C C I I CICI CIIC IICC ICIC ICCI	ICCC CICC CCIC CCCI	CCCC
Number of possible outcomes	1	4	6	4	1

PROPERTIES OF OUR QUIZ

- Two outcomes for each question **correct or incorrect**
- The probability of getting a correct answer is **0.5**
- The result for one question is **independent of the result of another**
- There is a fixed **number of questions (4)**

BIASED COIN MODEL **§5.3 PAGE 204 - 207**

Biased coin which has pr(H) = 0.6 . Tossed the coin 8 times. Counted the number of heads.

Recorded the results: { 6, 5, 5, 3, 7, 5, 8, ... }

Call this set of results X. I.e. X = number of heads in 8 tosses.

X has a **Binomial distribution** We write: X~ **Bin(8, 0.6)**

PROPERTIES OF THE BIASED COIN MODEL

- Two outcomes for each toss **head or tail**
- The probability of getting a head is **0.6**
- The result for each toss is **independent of the result of another toss**
- There is a fixed **number of tosses (8)**

Generally:

If pr(H) = p ; number of tosses = n X = number of heads in n tosses,

then $X \sim \text{Bin}(n, p)$

The Binomial Assumptions §5.3.2 page 205

If an experiment consists of a number of trials and

(i) each trial has only **2** outcomes: "success" or "failure"

(ii) the probability of "success", p, is the **same** for each trial | **Ie the conditions**
| **for each trial are**

(iii) the trials are **independent** | **identical**

(iv) X is the number of "successes" in a **fixed** number, n, of trials

then $X \sim \text{Bin}(n, p)$

Frivolous pursuits example continued

Assuming you do guess, the model for the number of correct answers in the quiz is

$X \sim \text{Bin}(n = 4,\ p = 0.5)$

Calculating Binomial Probabilities:

There are two types of computer output, take care!

1. Binomial (Individual Terms): $\text{pr}(X = x)$ Appendix A2 pages 559-561

If X~Bin(8, 0.6), then $\text{pr}(X = 5) =$ **0.2787**

If X~Bin(4, 0.5), then $\text{pr}(X = 2) =$ **0.3750** (Compare with previous example)

2. Binomial Distribution (Cumulative / Lowertail probabilities): $\text{pr}(X \le x)$

X~Bin(8, 0.6)

(i) $\text{pr}(X \le 3) =$

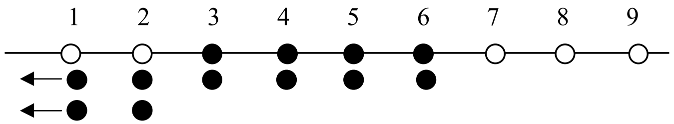
0.1737

(ii) $\text{pr}(X > 5) =$

1 – pr(X<=5) = 1 – 0.6846 = 0.3154

(iii)

$pr(3 \le X \le 6)$

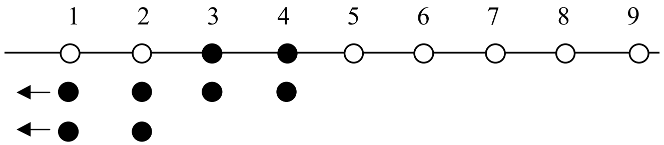


$pr(3 \le X \le 6) =$

pr(X<=6) – pr(X<=2) = 0.8936 – 0.0498 = 0.8438

(iv)

$pr(2 < X < 5)$



$pr(2 < X < 5) =$ **pr(X<=4) - pr(X<=2) = .4059 – .0498 = 0.3561**

Note: Using the Binomial formula or tables to find probabilities is non-examinable material. Computer output will be provided in tests and exams.

See: How to use MINITAB and *Excel* to produce probabilities page 207

See: Examples 5.3.1 & 5.3.2 page 207

Try: Exercises on §5.3 page 212

SAMPLING WITHOUT REPLACEMENT §5.3.3 page 210

If we take a sample of size n from a much larger population in which a proportion p have a characteristic of interest, the distribution of X, the number in the sample with that characteristic, is approximately Binomial (n, p)

(Operating rule: Approximation is adequate if $n/N < 0.1$; where N is the population size)

MEAN AND STANDARD DEVIATION OF THE BINOMIAL DISTRIBUTION §5.4 page 217-219

Frivolous pursuits example continued

Let X be the number of correctly guessed answers.

x	0	1	2	3	4
pr (X = x)	0.0625	0.25	0.375	0.25	0.0625

$\mu_X = E(X) =$ 0*.0625 + 1*.25 + 2 *0.375 + 3 *0.25 + 4*0.0625

= 2

$\sigma_X = sd(X) =$ $\sqrt{(0-2)^2 * 0.0625 + \ldots + (4-2)^2 * 0.0625}$

= $\sqrt{1}$ = 1

Suppose $X \sim \text{Bin}(n, p)$, then

$\mu_X = E(X) = np$ $\qquad$ $\sigma_X = sd(X) = \sqrt{np(1-p)}$

Frivolous pursuits example continued

$X \sim$ Binomial (n = 4, p = 0.5)

$\mu_x = E(X) = np = 4 * 0.5 = 2$

$\sigma_x = sd(X) = \sqrt{np(1-p)} = \sqrt{4 * 0.5 * 0.5} = \sqrt{1} = 1$

(compare with the 2 above)

See: Examples 5.4.3, 5.4.5 pages 217, 219

Try: Exercise 5.4.1, 5.4.3 pages 218, 220

Example:

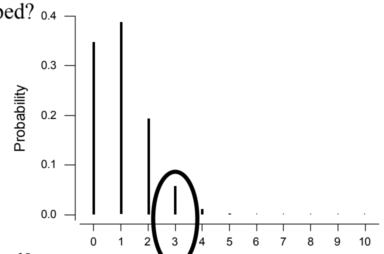
Suppose that 10% of bearings being produced by a machine have to be scrapped because they do not conform to the specifications of the buyer. In a batch of 10 randomly selected bearings:

Let X be the number of bearings to be scrapped from a batch of 10

$X \sim$ Bin (10, 0.1)

(a) what is the probability that exactly 3 have to be scrapped?

pr(X = 3) = 0.0574

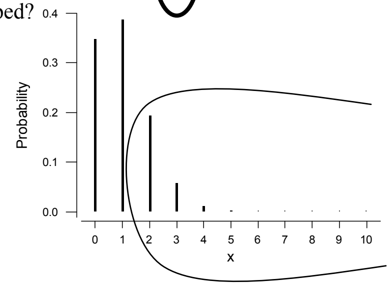


(b) what is the probability that at least 2 have to be scrapped?

pr(X >= 2) = 1 - pr(X <= 1)

= 1 – 0.7361

= 0.2639



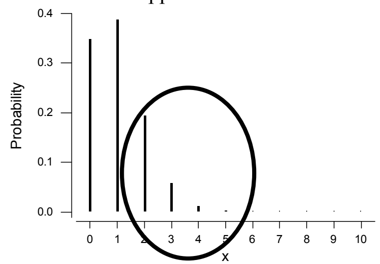
(c) what is the probability that at least 2 but no more than 5 have to be scrapped?

pr(2<=X <= 5)

= pr(X <= 5) - pr(X <= 1)

= 0.9999 – 0.7361

= 0.2638



(d) how many bearings would you expect to be scrapped?

E(X) = np = 10 * 0.1 = 1

(e) What is the standard deviation for the number of bearings scrapped in batches of 10 randomly selected bearings?

sd(X) = $\sqrt{np(1-p)} = \sqrt{10 * 0.1 * 0.9} = \sqrt{0.9}$ = 0.949

Try: Review Exercise 5 page 223

THE POISSON DISTRIBUTION §See notes following the end of this chapter.

Example:

An office receives, on average, 18 phone calls per hour. Count the number of calls received in one hour. On one occasion there were 13 calls in one hour. On another occasion there were 22 calls in one hour, and so on.

Let X be the number of phone calls per hour.

Under certain conditions we can say X has a **Poisson (18) distribution**

X ~ Poisson (18)

More examples:

X is the number of:

- errors in a given set of accounts.
- scratches on a cd surface
- bacterial colonies in 2 litres of milk

Ie **X is the number of occurrences in a given time/space under certain conditions**

Conditions:

1. The events occur at a constant average rate of λ (lambda) per unit time / space.
2. Occurrences are independent of one another.
3. The probability of 2 or more occurrences in a time / space interval of size d approaches 0 as d approaches 0. Ie Occurrences don't happen at *exactly* **the same time/space**

> If X is the number of occurrences per unit time / space and these occurrences obey the above conditions then X has a Poisson (λ) distribution.
>
> **X ~ Poisson (λ)**

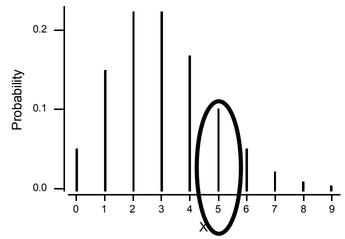
Notes:

(i) λ is the parameter for this distribution. (Where a parameter is a numerical characteristic of the population)

(ii) In this course it will **not** be necessary to use the formula for the Poisson probability function or tables. Computer output of pr($X = x$) (individual terms) and pr($X \le x$) (cumulative/lower tails) are given in tests and exams.

Example: If X ~ Poisson(λ = 3), find:

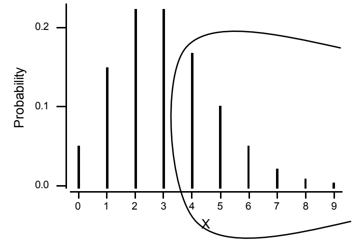
(i) pr(X = 5) = **0.1008**



(ii) pr(X > 3) = **1 – pr(X <=3)**

= 1 – 0.6472

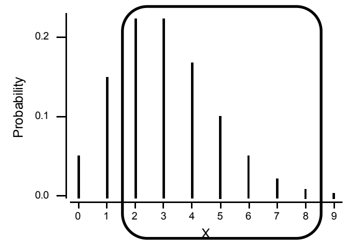
= 0.3528



(iii) pr(2≤ X≤ 8) = **pr(X <=8) – pr(X <=1)**

= 0.9962 – 0.1991

= 0.7971



Using *Excel*:

(i) pr(X = 5) = POISSON(5, 3, 0)

(ii) pr(X > 3) = 1- pr($X \le 3$) = 1 - POISSON(3, 3, 1)

(iii) pr($2 \le X \le 8$) = pr ($X \le 8$) - pr ($X \le 1$) = POISSON(8, 3, 1) - POISSON(1, 3, 1)

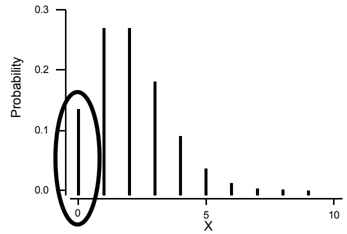
Example: Suppose a manufacturing company has a mean of 24 accidents per year. If the number of accidents per year is known to have a Poisson distribution, find the following probabilities.

Let X be the number of accidents per month

X ~ Poisson (λ=2) [λ=24/12]

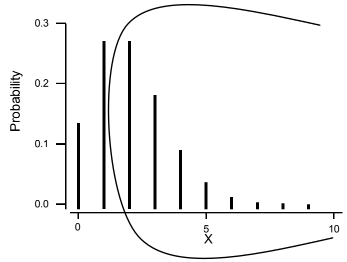
(i) There are no accidents in a given month.

pr(X = 0) = 0.1353



(ii) There are at least 2 accidents in a given month.

pr(X >= 2) = 1 - pr(X <= 1)
=1 - 0.4060
= 0.5940



MEAN AND STANDARD DEVIATION OF THE POISSON DISTRIBUTION

Suppose $X \sim$ Poisson(λ), then

$\mu_X = E(X) = \lambda$ $\sigma_X = sd(X) = \sqrt{\lambda}$

Rule of the thumb:

Usually more than 90% of observations from Poisson distributions will lie within 2 standard deviations of the mean.

Ie In the interval ($\lambda - 2\sqrt{\lambda}, \lambda + 2\sqrt{\lambda}$)

Example 1:

The Gainesville Sun (10 May 1980) reported that 18 employees of the nuclear weapons research facility, Lawrence Livermore Laboratory (California, U.S.A.), contracted the deadly skin cancer melanoma during the period 1972 - 1977. In comparison, in the previous 20-year period, the mean number of occurrences was 2.3 cases per 5-year period. If the mean rate of melanoma cases per 5-year period was 2.3, should it be considered unusual that 18 cases have occurred in one 5-year period?

Let X be the number of melanoma cases in a 5-year period. Then $X\sim$ **Poisson (λ = 2.3)**

2.3 +/- 2 * $\sqrt{2.3}$ = (-0.7, 5.3)

ie between 0 and 5

Ie 18 should be considered an unusually large number of cases of melanoma in a 5 year period. The situation has probably changed.

Example 2:

According to the New York Times, 5 February, 1985, officials in St. Petersburg, Florida were investigating the cause of death for 12 patients at a local nursing home in the two-week period November 13 - 26, 1984. According to the nursing home operator, 3 to 6 deaths per month would be "normal". Taking the larger of these figures to be the mean number of deaths per four-week period, should it be considered unusual that12 deaths per two-week period occurred?

Let X be the number of deaths in a 2-week period. Then $X\sim$ **Poisson (λ = 3)**

3 +/- 2 * $\sqrt{3}$ = (-0.5, 6.5)

ie between 0 and 7 deaths from natural causes. 12 deaths in a two-week period should be considered particularly unusual. The situation has probably changed.

Exercise

Identify whether the following random variables can be modelled by either the Binomial or Poisson distribution. If they can, find the value/s of the parameter/s.

(a) In the long run, 80% of ACE light bulbs last for 1000 hrs of continuous operation. You need to have 20 lights in your attic for a small business enterprise, so you buy a batch of 20 light bulbs. Let X_1 be the number of these bulbs that have to be replaced by the time 1000 hrs are up.

X_1 ~ Bin (20, 0.2)

(b) In (a), because of the continual need for replacement bulbs, you buy a batch of 1000 cheap bulbs. Of these 100 have disconnected filaments. You start off by using 20 bulbs (which we assume are randomly chosen). Let X_2 be the number with disconnected filaments.

X_2 ~ Bin (20, 0.1) (probability not constant, n/N = 20/1000 =.02 <.1)

(c) Suppose that telephone calls come into the university switchboard randomly at a rate of 100 per hour. Let X_3 be the number of calls in a 1-hour period.

X_3 ~ Poisson (100)

(d) In (c), 60% of callers know the extension number of the person they wish to call. Suppose 120 calls are received in a given hour. Let X_4 be the number of callers who know the extension number.

X_4 ~ Bin (120, 0.6)

(e) In (d), let X_5 be the number of calls taken up to and including the first call where the caller did not know the extension number.

Neither. Not fixed no of trials, not no of occurrences per unit

(f) It so happened that of the 120 calls in (d), 70 callers knew the extension number and 50 did not. Assume calls go randomly to telephone operators. Suppose telephone operator A took 10 calls. Of the calls taken by operator A, let X_6 be the number made by callers who knew the extension number.

X_6 ~ Bin (10, 70/120) (n/N = 10/120 = 0.08 < 0.1)

(g) Suppose heart attack victims come to a 200-bed hospital at a rate of three per week on average. Let X_7 be the number of heart attack victims admitted in one week.

X_7 ~ Poisson (3)

(h) Suppose 20 patients, of whom 9 had "flu", came to a doctor's surgery on a particular morning. The order of arrival was random as far as having flu was concerned. The doctor only had time to see 15 patients before lunch. Let X_8 be the number of flu patients seen before lunch.

Neither. Not Bin since n/N = 15/20 = 0.75 not < 0.1

(i) Suppose meteor showers are arriving randomly at a rate of 40 per hour. Let X_9 be the number of showers arriving in a 15-minute period. **X_9 ~ Poisson (10)**