Developing Statistical Thinking Handout

Workshop 1

Excerpt 1: *Fundamental Statistical Ideas in the School Curriculum and in Training Teachers (Burrill and Biehler 2011, p. 58)*

One perspective is provided by Wild and Pfannkuch's (1999) often quoted framework that focused on the thought processes involved in solving problems in statistics. The framework has four dimensions: investigative cycle, interrogative cycle, types of thinking, and dispositions. Within the types of thinking, those special to statistics are recognition of the need for data, transnumeration (changing representations of data to increase understanding), reasoning with statistical models, consideration of variation, and integrating statistics and context. The framework was not intended to illustrate how concepts develop across grade levels.

The Wild and Pfannkuch framework considers variability as the defining ingredient in statistical reasoning. They quote Snee (1990, p. 118), who defined statistical thinking as "thought processes, which recognise that variation is all around us and present in everything we do, all work is a series of interconnected processes, and identifying, characterising, quantifying, controlling, and reducing variation provide opportunities for improvement".

Excerpt 2: *Two tensions between school mathematical and statistical thinking (Burrill and Biehler 2011, pp. 64-65)*

Variation: Variation has a different nature in the two disciplines. Mathematics is often taught in school as being exact and precise. Statistics is about "noise", that is, how to measure and control variability. Real data in statistics are contextual, containing uncertainty and error while data in many school mathematics classrooms are typically assumed to perfectly fit a mathematical model. The teaching of functions in particular often undermines statistical concepts, for instance, when data lie exactly on a function graph.

Sampling and inference: As Freudenthal (1974) pointed out with regard to sampling: what is important for statistics is sample-to-sample variation and how this variation decreases as the sample size increases. An intuitive understanding of this property can prevent students from believing in the law of small numbers, an unrealistic stability of samples with "small" sample sizes (Tversky & Kahnemann, 1971). The mathematical approach to proportional reasoning, however, often undermines the statistical approach for reasoning from samples. Percentages in mathematics are often applied in simple contexts, where the reference is set and the units are clear and constant. Careful statistical statements made about margin of error and confidence intervals are replaced by simplistic "inferences" from "sample" to "population", assuming a perfect proportional relationship. Ignoring uncertainty and variability, sample results are reported in point estimates rather than interval estimates in many media reports. Preparing students for statistical thinking requires that discussions in mathematics classrooms make this difference explicit.

Excerpt 3: *Inference (Burrill and Biehler 2011, p. 65)*
With regard to inference there are the following tensions. In mathematics, deciding what to believe is straightforward: conclusions follow deductively from definitions and agreed-on principles. In statistics, reasoning is partly inductive, and conclusions always ***uncertain.*** The degree of faith in a statistical conclusion depends on the integrity of the entire investigative process, while in mathematics a proof makes you ***certain.*** In statistics, how the data were collected and the role of randomness determines how you can interpret the results, while in (pure) mathematics, the reasoning is independent of the data. However, justifying the validity of mathematical models requires reasoning more akin to statistical reasoning than to reasoning in pure mathematics.

Excerpt 4: *Teaching Statistics in the Mathematics Classroom (Gattuso and Ottaviani 2011, p. 124)*
In addition to gaps in teachers' statistical knowledge, negative attitude and beliefs towards statistics complicate the situation. "Negative attitudes are linked to perceived difficulty, lack of knowledge and overly formal learning experience" (Estrada & Batanero, 2008, p. 5). Meletiou (2003) argued that beliefs about the nature of mathematics affect instructional approaches and curricula in statistics, and act as a barrier to the kind of instruction that would provide students with the skills

necessary to recognise and intelligently deal with uncertainty and variability. Although the teaching of mathematics has undergone many changes and proposes a constructivist approach, long-held beliefs and attitudes of teachers are difficult to change. Statistical concepts linked to context should be approached as social constructs, following the way suggested by the data-oriented approach. In reality, concepts are too often presented to students without any links to the real-world context or at the most within artificial examples and using a traditional and procedural approach that in many cases meet students' and parents' expectations.

Excerpt 5: The Nature of Mathematics: Towards a Social Constructivist Account (Ernest, 1992, pp. 99-100)

It is appropriate to indicate, however briefly, some of the educational implications of the social constructivist account of mathematics….One aspect of this view is that mathematics is seen as embedded in a cultural context. It leads to the conclusion that the view that mathematics somehow exists apart from everyday human affairs is a dangerous myth. It is dangerous, not only because it is philosophically unsound, but also it has damaging results in education….On the other hand, if mathematics is viewed as a social construct, then the aims of teaching mathematics need to include the empowerment of learners to create their own mathematical knowledge; mathematics can be reshaped, at least in school, to give all groups more access to its concepts, and to the wealth and power its knowledge brings; the social contexts of the uses and practices of mathematics can no longer be legitimately pushed aside, the uses and implicit values of mathematics need to be squarely faced, and so on…This second view of mathematics as a dynamically organised structure located in a social and cultural context, identifies it as a problem posing and solving activity. It is viewed as a process of inquiry and coming to know, a continually expanding field of human creation and invention, not a finished product. Such a dynamic problem solving view of mathematics embodied in the mathematics curriculum, and enacted by the teacher, has powerful classroom consequences. In terms of the aims of teaching mathematics the most radical of these consequences are to facilitate confident problem posing and solving; the active construction of understanding built on learners' own knowledge; and the exploration and autonomous pursuit of the learners' own interests.

If mathematics is understood to be a dynamic, living, cultural product, then this should also be reflected in the school curriculum. Thus mathematics needs to be studied in living contexts which are meaningful and relevant to the learners. Such contexts include the languages and cultures of the learners, their everyday lives, as well as their school based experiences. If mathematics is to empower learners to become active and confident problem solvers, they need to experience a human mathematics which they can make their own. The social constructivist view places a great deal of emphasis on the social negotiation of meaning. Clearly this has very strong implications for discussion in the mathematics classroom.

The social constructivist view also raises the importance of the study of the history of mathematics, not just as a token of the contribution of many cultures, but as a record of humankind's struggle - throughout time - to problematise situations and solve them mathematically - and to revise and improve previous solution attempts. By legitimating the social origins of mathematics, this view provides a rationale, as well as a foundation for a multicultural approach to mathematics.

Excerpt Sources

Burrill, G., & Biehler, R. (2011). Fundamental statistical ideas in the school curriculum and in training teachers. In C. Batanero, G. Burrill, & C. Reading (Eds.) (Vol. 14, pp. 57–69). Netherlands: Springer.

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