

Discrete

Continuous
Probability is AREA

Continuous =
Measured:
time, weight,
length, etc

Uniform

Only “known” things
are highest and lowest
values. No reason to
assume values are not
equally likely.

Triangular

As well as maximum
and minimum, a MOST
LIKELY value is known

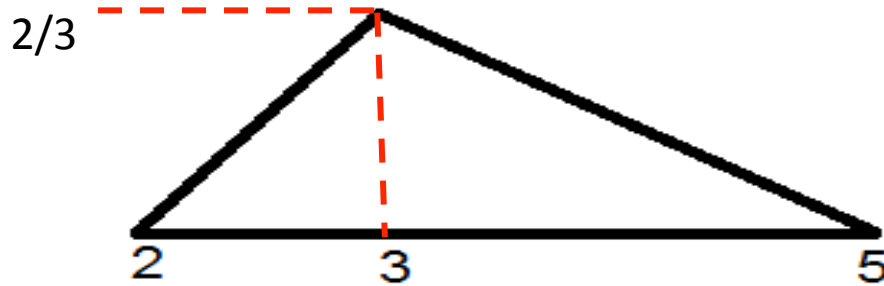
Normal

Question usually says
Normally-Distributed!

Mean and SD
mentioned

Triangular distribution diagram setup

A parcel will take somewhere **between 2 and 5** hours to be delivered. It is **most likely** to be delivered in **3 hours**.



Width of triangle = 3

Area must be 1.

$\frac{1}{2}$ base x height = 1

$\frac{1}{2}$ of 3 x height = 1

Height = $1 / 1.5 = 2/3$

In general, height = $1 \div (\text{HALF base})$ or $2 \div \text{base}$

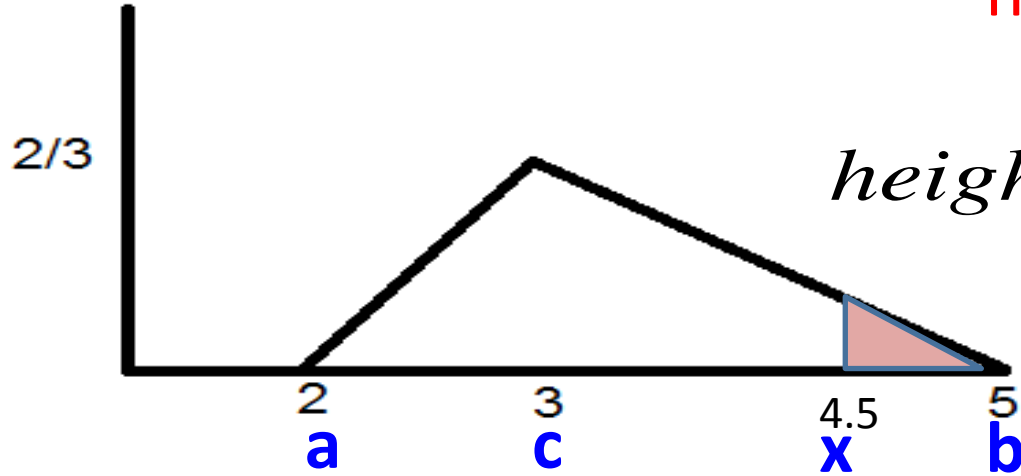
The height at any point on the triangle is given by this formula

$$f(x) = \begin{cases} 0, & x < a \\ \frac{2(x-a)}{(b-a)(c-a)}, & a \leq x \leq c \\ \frac{2(b-x)}{(b-a)(b-c)}, & c \leq x \leq b \\ 0, & x > b \end{cases}$$

Where a is lowest, c is peak, b is highest, x is the edge of the shading

What is the probability that the parcel will take longer than 4.5 hours?

Use the formula to find the height of the pink triangle.



$$\text{height} = \frac{2(5 - 4.5)}{(5 - 2)(5 - 3)} = \frac{1}{6}$$

$$f(x) = \begin{cases} 0, & x < a \\ \frac{2(x-a)}{(b-a)(c-a)}, & a \leq x \leq c \\ \frac{2(b-x)}{(b-a)(b-c)}, & c \leq x \leq b \\ 0, & x > b \end{cases}$$

x is in between c and b

$$\begin{aligned} \text{Area} &= \frac{1}{2} \times \text{base} \times \text{height} \\ &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{6} \\ &= \frac{1}{24} = 0.042 \end{aligned}$$

Steps to take in a triangle distribution

- Draw the triangle and calculate the height.
- Shade the area being described
- IDENTIFY THE TRIANGLE/S YOU CAN USE.
- Use the formula to find the heights of the triangles you can use.
- Calculate area of small triangle.
- *Finding the shaded area may involve a subtraction*

How would you find these areas?

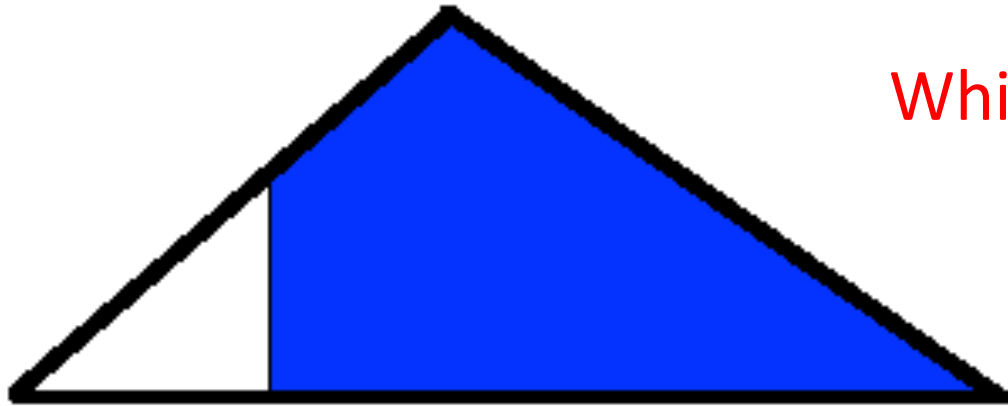


You want the blue section

It's already a triangle

Answer will be
area of blue triangle

How would you find these areas?



You want the blue section

White area is a triangle

Calculate white area

Answer will be

$1 - \text{white area}$

How would you find these areas?



These are often Excellence!

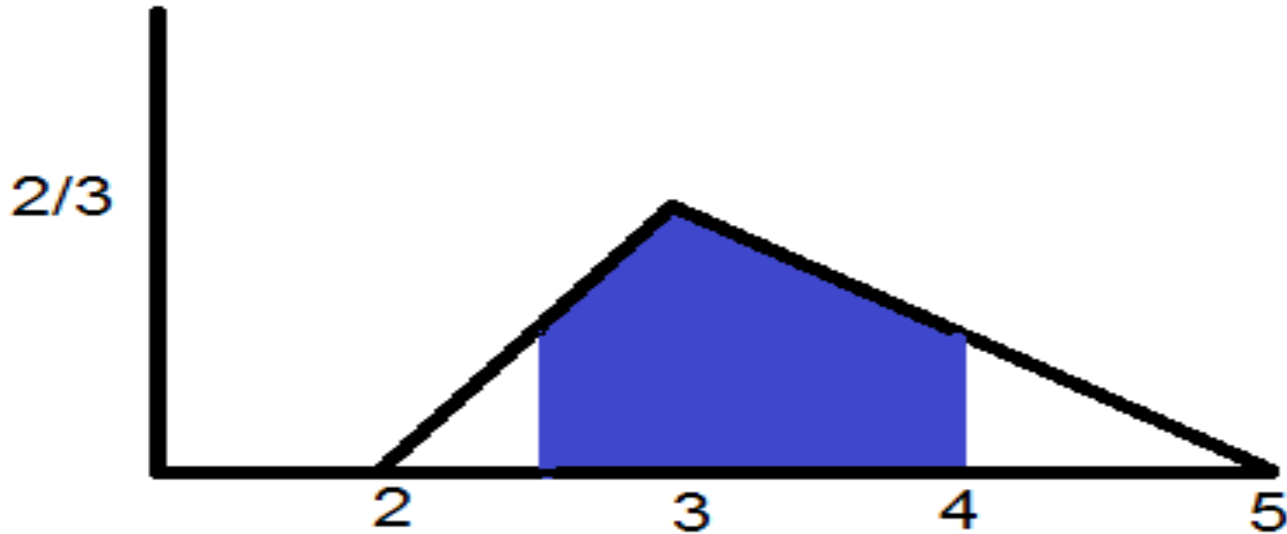
You want the blue section

White triangles can be
calculated

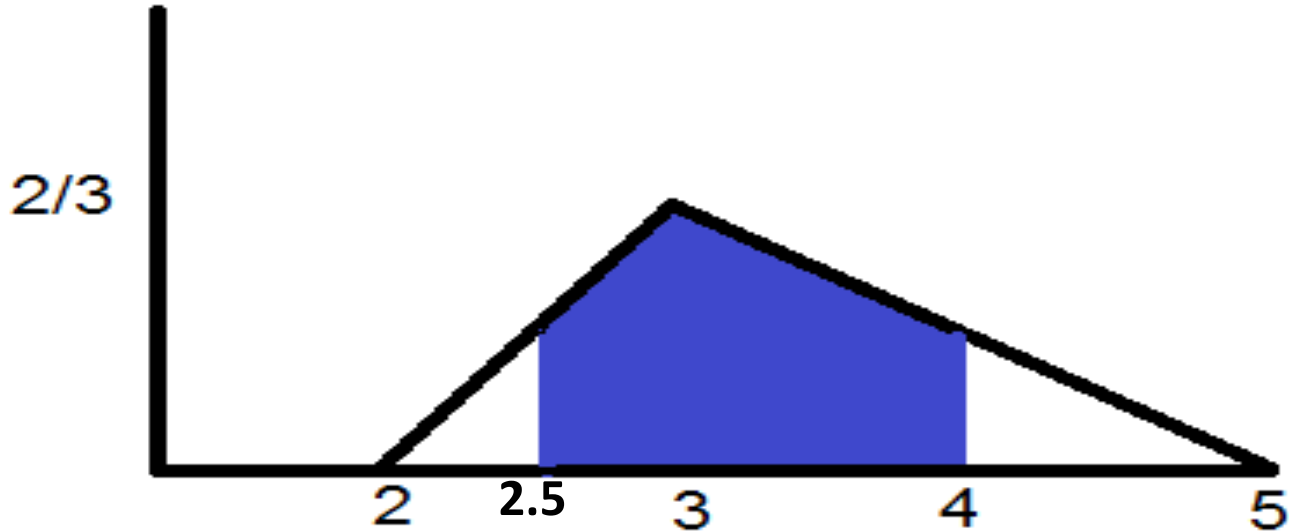
Answer will be
1 – BOTH white areas

Probability Q use similar triangles

Nasty one: What is the probability that the parcel takes between 2.5 and 4 hours?

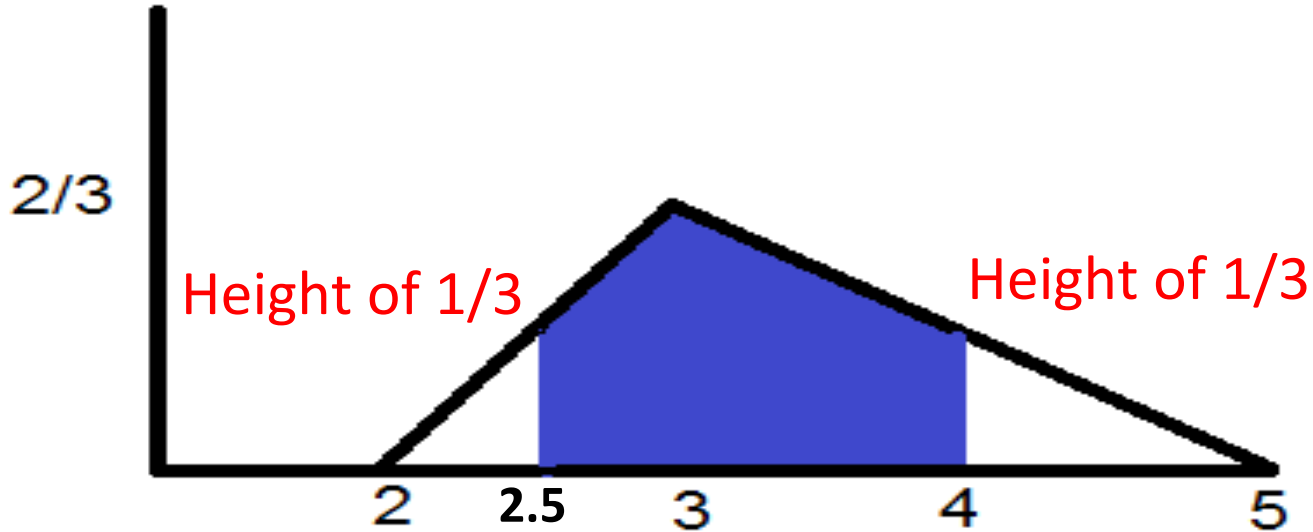


Working:



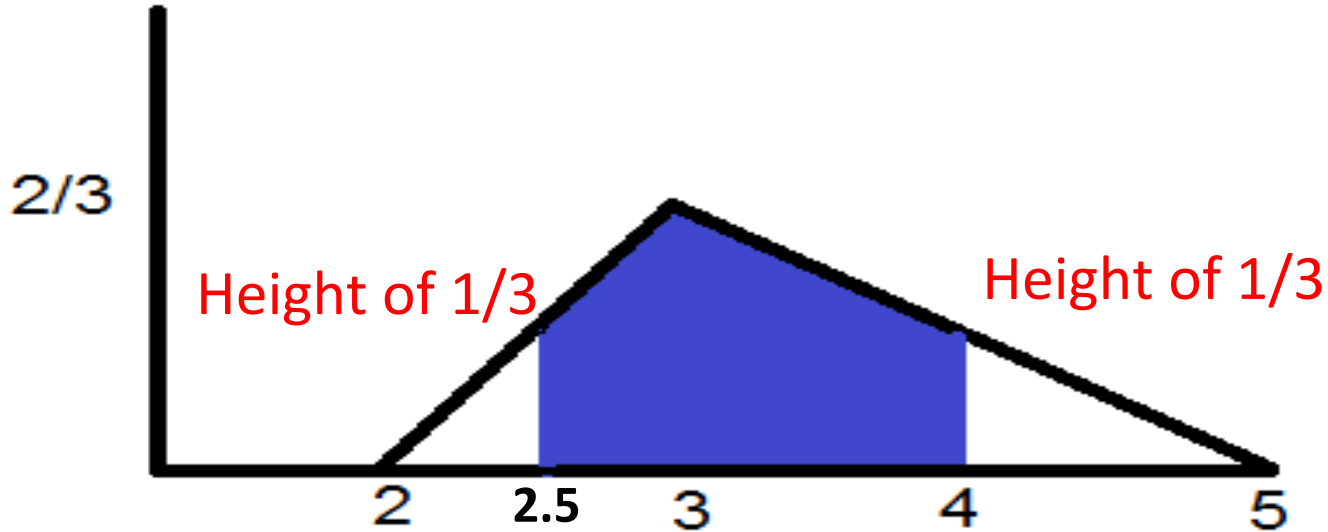
Need to subtract the two small white triangles from 1 (the total area). This will leave the blue shaded area

Working:



Heights are calculated using the formula

Working:



$$\begin{aligned} \text{Area} &= 1 - \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{3}\right) - \left(\frac{1}{2} \times 1 \times \frac{1}{3}\right) \\ &= 1 - \frac{1}{12} - \frac{1}{6} = 0.75 \end{aligned}$$

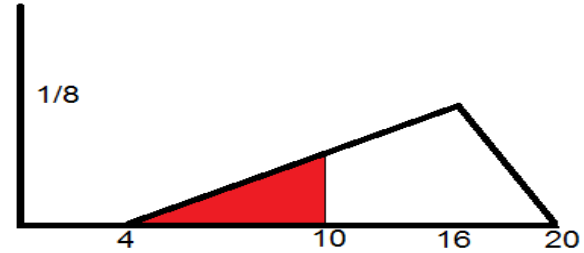
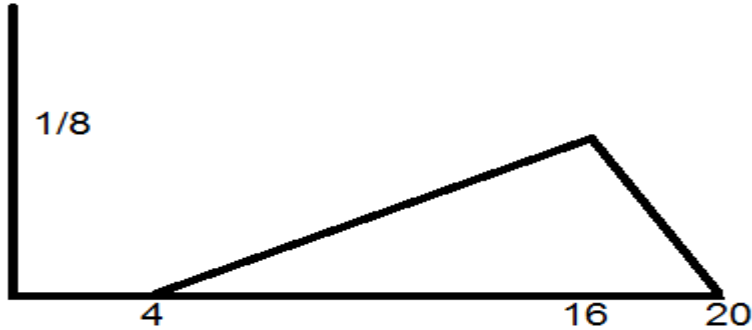
Try these:

It is known that a certain cat will sleep between 4 and 20 hours in a day, with the most likely time being 16 hours

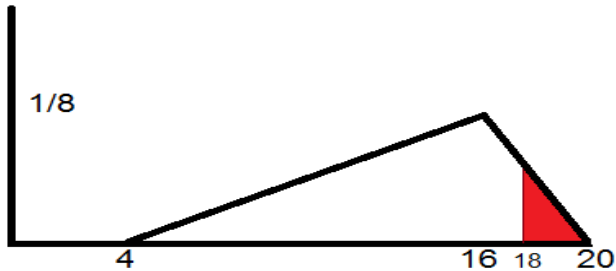
1. Draw the probability diagram. Work out the height
2. Use this to calculate the probability of a cat sleeping
 - (a) Less than 10 hours
 - (b) More than 18 hours
 - (c) 6 – 16 hours
 - (d) 10 – 17 hours



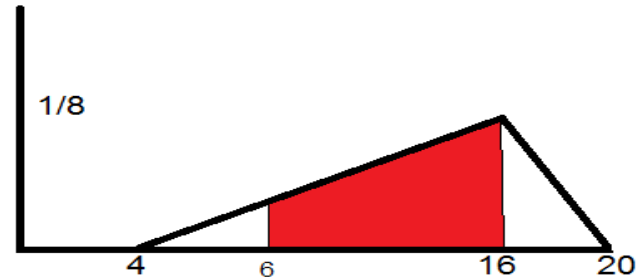
Answers



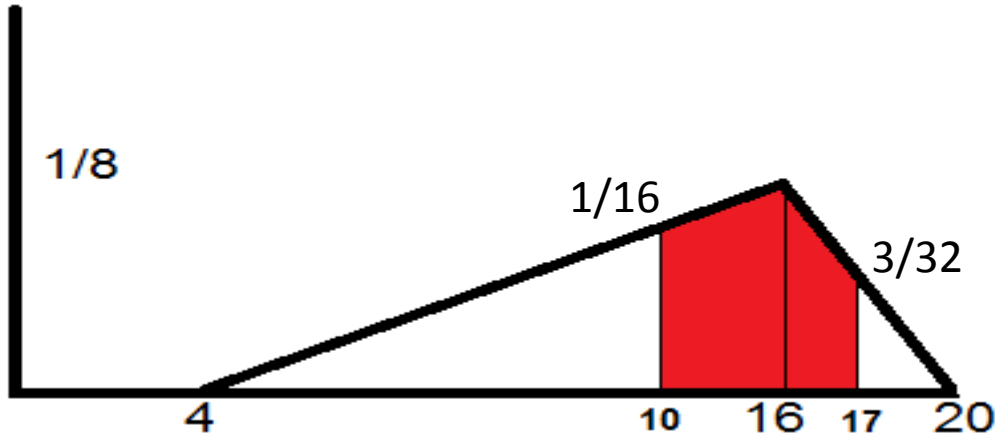
$$\text{Area} = 0.5 \times 6 \times 1/16 = 0.1875$$



$$\text{Area} = 0.5 \times 2 \times 1/16 = 0.0625$$



Area of triangle from 4-16 minus area of triangle from 4 – 6 = 0.73 (2 d.p.)



Area of triangle from 4 to 16 minus area of triangle from 4 to 10
 PLUS

Area of triangle from 16 to 20 minus area of triangle from 17 to 20
 $= \frac{1}{2} \text{ of } 12 \times \frac{1}{8} - \frac{1}{2} \text{ of } 6 \times \frac{1}{6} + \frac{1}{2} \text{ of } 4 \times \frac{1}{16} - \frac{1}{2} \text{ of } 3 \times \frac{3}{32}$
 $= 0.234$