Triangular Distribution

The Triangular Distribution is often used as a subjective description of a population. It is based on estimates of the minimum and maximum and an inspired guess as to what the modal value might be. Despite being a simplistic description of a population, it is a useful distribution for modelling subjective probability where hard data is scarce (possibly because of the high cost of collection). One way of obtaining the the min, mode and max parameters is to ask the opinion of someone with appropriate experience. The human mind is good at reducing complex datasets to manageable proportions!

Triangular distributions are often used in oil and gas exploration where data is expensive to collect and it is almost impossible to model the population being sampled accurately, thus subjectivity plays a greater role than in data rich sectors.

Probabilities are calculated by finding areas of triangles. You will need to use properties of similar triangles to do this.

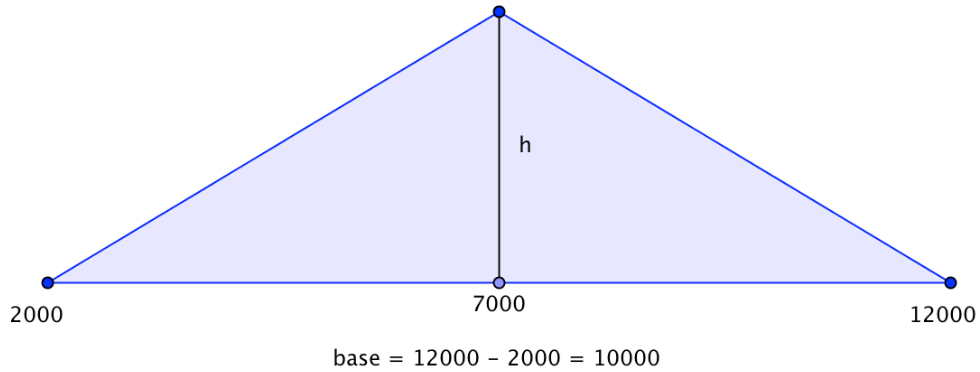
Example

An oil company expects that a new oil field off the East Cape of New Zealand will generate between 2000 and 12000 barrels of oil per hour. The oil company also expects the modal number of barrels per hour to be 7000 barrels per hour.

There has been considerable public protest about this oil field. The oil will be extracted using fracking and the local people are concerned that this will increase the likelihood of earthquakes in the East Cape. Despite this the company has been given a licence to explore the possibility of extracting oil from this field. The company believes the oil field will be uneconomic if the probability of extracting less than 5000 barrels per hour is less than 0.1

a. Model the hourly production of oil by a triangular distribution.
b. Find the probability that less than 5000 barrels of oil will be produced in one hour.
c. Find the probability that less than 5000 barrels of oil are produced in exactly 18 hours of a 24 hour day.

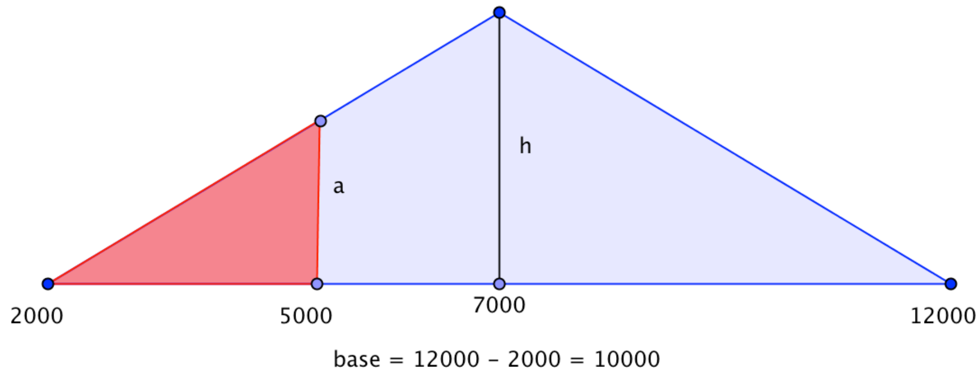
a The distribution of hourly oil production can be modelled by a triangular distribution. The base of the triangle lies between 2000 and 12000 barrels. The vertex of the triangle lies at 7000 barrels. We can calculate the area of the triangle by making the area of the triangle 1.



The area of this triangle is 1.

$\frac{1}{2} \times 10000 \times h = 1$ $h = 2/10000 = 0.0002$

b To solve the first problem we need the area of the red triangle. First we need to find the height using similar triangles. The two triangles we use are the red triangle and the left hand half of the original distribution



The two heights of the triangles are a and h
The two bases of the triangles are 3000 and 5000
Because the triangles are similar the ratios are equal.

$$\frac{a}{h} = \frac{3000}{5000}$$

$$a = 3000\ x\ \frac{h}{5000} = 3000\ x \frac{0.0002}{5000} = 0.00012$$

The area of the red triangle is 0.5 x 3000 x 0.00012 = 0.18

The probability of the oil field producing less than 5000 barrels per hour is 0.18

c The second problem is now a binomial calculation

P(X = 18) when n = 24 and p = 0.18

You can calculate this using BPD on your calculator.

Problems

1. A company's revenues are projected to be $50 million in some future year t. Current annual revenue is $10 million. We can say with some conviction that gross revenue in year t will not be less than $10 million and will not be greater than $90 million.

Using a triangular distribution what are the probabilities that

a. Sketch the distribution
b. Find the probability that the revenue in year t will be less than $30 million and
c. Find the probability that the revenue in year t will be greater than $60 million?

2. Anna babysits for a family. She knows that the length of time that she babysits when the parents go out for a meal is between 90 and 140 minutes. Usually the length of time is 115 minutes.

Model this by a triangular distribution.

a. Sketch the distribution
b. Find the probability Anna is required to babysit one evening for less than 100 minutes
c. Find the probability Anna is required to babysit one evening for between 100 and 130 minutes.

d. Anna babysits for 5 evenings. Find the probability that she is required for between 100 and 130 minutes on 3 evenings out of 5.

3. The length of a school staff professional learning (PLD) session is always more than 50 minutes and never longer than 80 minutes. Usually they last 60 minutes.

Model this by a triangular distribution

a. Sketch the distribution
b. Find the probability a PLD session lasts for more than 60 minutes.
c. Find the probability a PLD session lasts 60 minutes to the nearest 5 minutes.
d. There are 6 PLD sessions a term. Find the probability that at least 4 of these last more than 60 minutes.

4. Sarah is starting up a new business and is selling coasters for wine glasses. These are a miniature jandal and look rather neat.

She expects her weekly sales in the start up phase to be at least 100 and at most 1100. Market research indicates that 600 is the most likely number of sales.

a. Model this by a triangular distribution.
b. Find the probability that a weeks sales will be more than 500.
c. Find the probability that a week's sales will be more than 800.
d. The initial marketing phase is for ten weeks. Find the probability that there will be more than 800 sales in at least 4 of these weeks.

5. Emelia is launching a new fashion weekly magazine. She expects that at least 200 copies and at most 800 copies will be bought each month in the Auckland region in the first few months. 500 is expected to be the modal number of sales

a. Model this by a triangular distribution
b. Find the probability that the first months sales will be more than 600 copies.
c. The magazine is published for six months initially. Find the probability that there will be at least 600 copies sold in at least 4 of these six months,