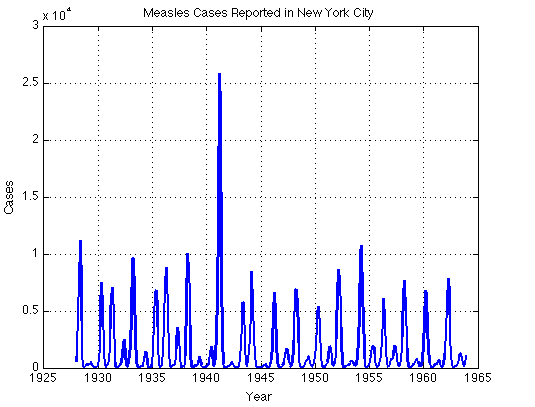
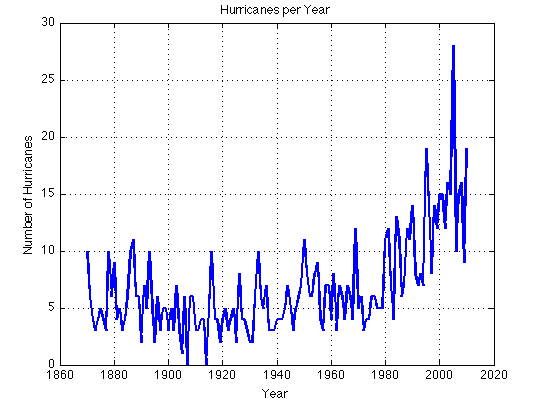
Appendix 1

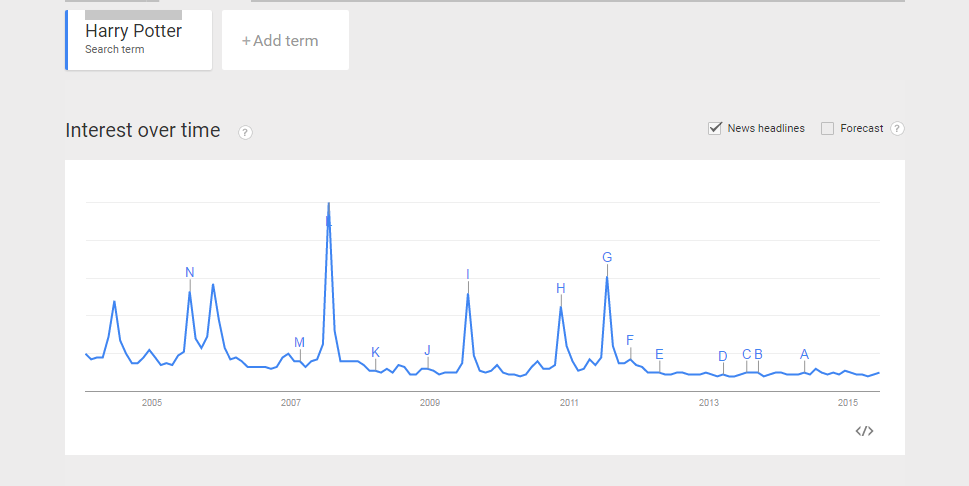
TIME SERIES – TREND DESCRIPTION

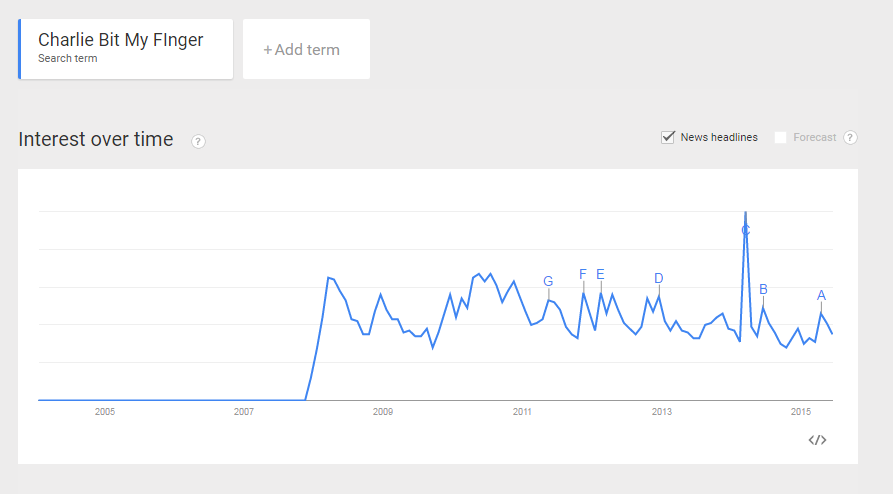






FROM GOOGLE TRENDS





Appendix 2

EXAMPLES OF SMOOTHING TECHNIQUES FOR TIME SERIES

STUDENT INVESTIGATION INSTRUCTIONS

Introduction

There are many different ways to smooth a time series. Methods depend on the type of time series but have also depended historically on ease of calculation. Thus historically the Moving Mean or Centred Moving Mean approach has been favoured for its ease of calculation. Technological change means we are no longer restricted to such a smoothing technique especially given its disadvantages.

Techniques to be investigated

1. Moving Mean
2. Weighted Moving Mean
3. Exponential smoothing

INSTRUCTIONS

1. Copy EXCEL data file called LArain. This contains data on rainfall in Los Angeles, measured in inches between 1908 and 1973. The time variable should be in column A and the rainfall data in column B. Add these headings in the suggested cells

D1 – enter MM(3) or Moving Mean , order 3

F1 – enter Weighted Moving Mean

H1 – enter Exponentially smoothed ( α = 0.5)

K1 = enter Exponentially smoothed ( α = 0.1)

1. Plot the time series using INSERT option on EXCEL
2. **SMOOTHING TECHNIQUE 1** – This technique smooths the series by calculating the mean of number of consecutive values of the time series. The ORDER of the moving mean refers to the number of consecutive values you include in the calculation of your mean.

Eg ORDER = 3 means

First smoothed value = mean of first three values of series

This first smoothed value will be plotted against the second time period in your series.

1. In cell D3 enter the following formula

=average(B2:B4) & then copy formula down to cell D66.

1. Plot LA rainfall and Moving Mean (Order 3) on same graph.
2. **SMOOTHING TECHNIQUE 2** – The moving mean technique applies an equal weight to each of the previous values included in the smoothing calculation. Smoothing using a weighted mean allows us to manipulate these weights. For example, instead of using equal weights we could allocate 50% (or 0.50) weighting to the most recent value, 30% (or 0.30) weighting to the value before that and 20% (or 0.20) weighting to the value before that.

Eg First smoothed value = (0.5 x third value) + (0.3 x second value) + (0.2 x first value)

1. In cell F5 enter the following formula

=0.5\*B4+0.3\*B3+0.2\*B2 & copy formula down to cell F67

1. Plot LA rainfall data and Weighted Moving Mean on same graph.
2. **SMOOTHING TECHNIQUE 3** - With exponential smoothing, each smoothed value is a weighted mean of ALL of the previous values in the series. The weights decrease in size over time. Greater weight is attached to more recent values; less weight is attached to values further in the past. Exponential smoothing requires TWO initial parameters –

First smoothed value = estimate by first data value

Smoothing parameter, α, which can range 0 < α < 1, but is usually below 0.5.

Enter 0.5 in cell I2.

1. Insert following formula in cell H3

= B2

This initializes the first smoothed value to first data value

1. Insert the following formula in cell H4

=$I$2\*B3+ (1-$I$2)\*H3

This calculates the second smoothed value. The $ signs surrounding I2 can be added by pressing F4. This ensures that the constant, α, does not change when the formula is copied.

Copy the formula down to cell H67

1. Plot LA rainfall data and exponentially smoothed series on same graph.
2. **SMOOTHING TECHNIQUE 4** – This is another exponential smoothing technique, but this time we are going to reduce the smoothing parameter, α, from 0.5 to 0.1.
3. Initialise the first smoothing value. Insert the following formula in cell K3

= B2

1. Initialise the smoothing parameter, α.

Enter 0.1 in cell L2

1. Insert the following formula in cell K3

=$L$2\*B3+(1-$L$2)\*K3 in cell K4 and then copy to cell K67. Again use F4 button to keep value of α unchanged in the formula.

1. Plot LA rainfall and exponentially smoothed series on same graph.

QUESTION – WHICH TECHNIQUE DO YOU PREFER AND WHY?

EXTRA ACTIVITIES TO TRY

1. Adjust order of Moving Mean. What does an order 4, 5 or bigger look like? What are the disadvantages of a higher order Moving Mean?
2. Adjust the weights in the Weighted Moving Mean. What difference would weights of 70%, 20% & 10% look like for example? Try some other weights or perhaps another order and another set of weights.
3. Adjust values of α. Try values in between 0.4, 0.3 or 0.2.
4. Repeat exercise with another stationary ( no trend) series.

**Appendix 3**

**A TEACHER’S GUIDE TO THE MODELS USED IN TIME SERIES MODULE OF iNZight**

**Introduction**

The Time Series module of the FREE software package iNZight uses two different statistical models. The model used to obtain the series decomposition is called a Seasonal Trend Lowess and the model used to calculate predictions is a Holt-Winters model. This guide is to give teachers a brief summary of the models used but the new standard has no expectation that students need to know any theoretical background to the models.

**Seasonal Trend Lowess**

**(LOWESS – Locally Weighted Regression Scatterplot Smoothing)**

Smoothing or filtering a Time Series is best thought of as similar to the idea of filtering music through an amplifier. We can amplify certain sounds or we can suppress certain sounds. Similarly, we can suppress (remove) certain features in a Time Series, such as seasonality, in order to model the trend and/or cycle. Once we have built a suitable model for the smoothed series, we can add back the appropriate seasonal component in order to produce predictions.

A common method for smoothing a Time Series is to use moving averages, which is what has traditionally be taught in schools for AS 3.1. One drawback of moving averages is that our moving average series becomes shorter than the original Time Series. If we have monthly data, our first moving average value is calculated on observations 1 to 12, and the second moving average value is calculated on observations 2 to 13. We then average these two values to get our first moving average value which then replaces observation 7 in our original series. Similarly, at the end of our series, there are six observations that we have no moving average values for.

A more useful tool for isolating and then removing the seasonal component of a Time Series is **Seasonal Trend Lowess** a decomposition function in R ( the programming language that iNZight iswritten in*)*. The method used is to first smooth the trend and cycle using a lowess smoother (fitting a local regression to a window of points and using the point on the fitted regression line as the value of the smooth for the time value in the middle of the window). The regression that is used is “weighted”, in that observations near the edge of the window are given less weight than observations near the centre of the window when determining the local regression line. Then a separate lowess smoother is used on each seasonal sub-series (i.e. all the January observations, all the February observations, …). The “trend and cycle” smoothed value and the appropriate “seasonal” smoothed value can be subtracted from the original observation to yield the remainder or random component for that observation. iNZight produces a plot of the decomposition that shows the original series, the seasonal component, the trend and cycle and finally, the random component.

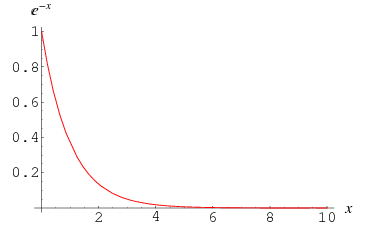
A third option for smoothing data is exponential smoothing and it is this technique that is used in the Holt-Winters model.

**Holt-Winters Model**

This model, often referred to as a procedure, was first proposed in the early 1960s. It uses a process known as exponential smoothing. All data values in a series contribute to the calculation of the prediction model.



**Exponential smoothing** in its simplest form should only be used for non-seasonal time series exhibiting a constant trend (or what is known as a stationary time series). It seems a reasonable assumption to give more weight to the more recent data values and less weight to the data values from further in the past. An intuitive set of weights is the set of weights that decrease each time by a constant ratio. Strictly speaking this implies an infinite number of past observations but in practice there will be a finite number. Such a procedure is known as **exponential smoothing** since the weights lie on an exponential curve.



If the smoothed series is denoted by *St*

*α* denotes the smoothing parameter, the exponential smoothing constant, 

The smoothed series is given by: *St* = *α yt +* (1 *- α*)*St-*1

where *S*1 = *y*1

The smaller the value of *α*, the smoother the resulting series.

It can be shown that: *St* = *α* *yt* + *α*(1  *α*)*yt-*1 + *α*(1  *α*)2*yt*-2 + …+ (1  *α*)*t*-1 *y*1

Consider the following Time Series:

14 24 5 18 10 17 23 17 23 …

Using the formulae above, with an exponential smoothing constant, *α* = 0.1

*S*1 = *y*1 = 14

*S*2 = *α y*2 + (1 - *α*)*S*1 = 0.1(24) + 0.9(14) = 15

*S*3 = *α* *y*3 + (1 - *α*)*S*2 = 0.1(5) + 0.9(15) = 14

*S*4 = *α* *y*4 + (1 - *α*)*S*3 = 0.1(18) + 0.9(14) = 14.4

etc

Thus the smoothed series depends on all previous values, with the most weight given to the most recent values.

Exponential smoothing requires a large number of observations.

Exponential smoothing is not appropriate for data that has a seasonal component, cycle or trend. However, modified methods of exponential smoothing are available to deal with data containing these components.

The Holt-Winters model uses a modified form of exponential smoothing. It applies three exponential smoothing formulae to the series. Firstly, the level (or mean) is smoothed to give a local average value for the series. Secondly, the trend is smoothed and lastly each seasonal sub-series ( ie all the January values, all the February values….. for monthly data) is smoothed separately to give a seasonal estimate for each of the seasons. A combination of these three series is used to calculate the predictions output by iNZight.

The exponential smoothing formulae applied to a series with a trend and constant seasonal component using the Holt-Winters additive technique are:







where:

*α*, *β* and *γ* are the smoothing parameters

a*t* is the smoothed level at time *t*

b*t* is the change in the trend at time *t*

s*t* is the seasonal smooth at time *t*

*p* is the number of seasons per year

The Holt-Winters algorithm requires starting (or initialising) values. Most commonly:







The Holt-Winters forecasts are then calculated using the latest estimates from the appropriate exponential smooths that have been applied to the series.

So we have our forecast for time period :



where:  is the smoothed estimate of the level at time *T*

 is the smoothed estimate of the change in the trend value at time *T*

 is the smoothed estimate of the appropriate seasonal component at *T*

As mentioned earlier the Holt-Winters model assumes that the seasonal pattern is relatively constant over the time period. Students would be expected to notice changes in the seasonal pattern and identify this as a potential problem with the model, particularly if long–term predictions are made. In practice this is dealt with by transforming the original data and modelling the transformed series or using a multiplicative model. Students are not expected to know this, but are required to identify a variable seasonal pattern as a potential problem. The exponential smoothing formulae applied to a series using Holt-Winters Multiplicative models are:







The initialising values are as for the additive model, except:



So we have our prediction for time period :



Calculation of Prediction Intervals for Holt Winters

**Reference Yar,M. & Chatfield, C. ( 1990) Prediction intervals for the Holt-Winters forecasting procedure, *International Journal of Forecasting*, Vol. 6,pp 127-137, North Holland.**

There are many situations where it is important to give interval predictions, rather than point predictions, as a means of assessing future uncertainty. An interval prediction associated with a prescribed probability is sometimes called a confidence interval, but it is recommended that the term prediction interval is used in the context of time series analysis. This is because prediction interval is more descriptive and because the term ‘confidence interval’ is usually applied to interval estimates of model parameters. Unfortunately it is relatively common to see predictions made without any reference to prediction intervals. This may be because there are a number of different ways that prediction intervals can be calculated. The paper above provides not only details of how the prediction intervals for Holt-Winters are produced but also compares the authors’ preferred method with several alternative methods. It also compares the prediction intervals calculated for the same data set by a variety of different models.

Derivation and details of prediction interval calculation can be found in Yar & Chatfield’s (1990) paper – see section 3 and 4 on page 129.

In one example given in the paper, a monthly index of employment in manufacturing in Canada, a prediction for three years after the end of the actual data is provided of 115.9. A prediction interval of [113.95,117.85] is also calculated. A suggested interpretation of this prediction interval (P.I.) is

‘ There is a 95% chance that the true index value of employment in manufacturing in Canada in three years time will be between 113.95 and 117.85.’

The details given in this paper apply to an additive Holt-Winters model only.

Assessing non-stationary model forecasts

The test of any prediction model is how well does it predict when compared to actual data values. To do this either remove the last few given observations or find the next few actual observations. Different prediction models can then be compared using a statistic known as the Root Mean Squared Error of Prediction (RMSEP). The formula for calculating this statistic is given below



where *τ* is the number of predictions we are using in the calculation of *RMSEP*.

Students are not expected to calculate RMSEP.

**Want to know more?** There are several Youtube clips that explain exponential smoothing and Holt-Winters using Excel or R if you are interested.

**Summary of what students need to know**

* Holt –Winters Additive model assumes seasonal pattern is reasonably constant
* Holt-Winters Model uses a technique of exponential smoothing, which is a weighted sum of previous values in a series. More weight is given to more recent values and less weight is given to values from the distant past.
* Holt-Winters Additive model exponentially smooths three series in order to produce predictions – the level, the trend and the seasonal sub-series.
* Students should be able to identify cyclical components and inconsistent seasonal patterns. They should note that such features are incompatible with assumptions underlying Holt-Winters Additive model and suggest a multiplicative model be considered instead. Such a comment would be expected at Excellence level only. Students are NOT expected to calculate a multiplicative model.

With thanks to Mike Forster, Department of Statistics, University of Auckland