Mathematics & Statistics Faculty

Michelle Dalrymple

Use Statistical methods to make a formal inference

The end of the high school journey…

Final step in the sample→ population inference journey for students

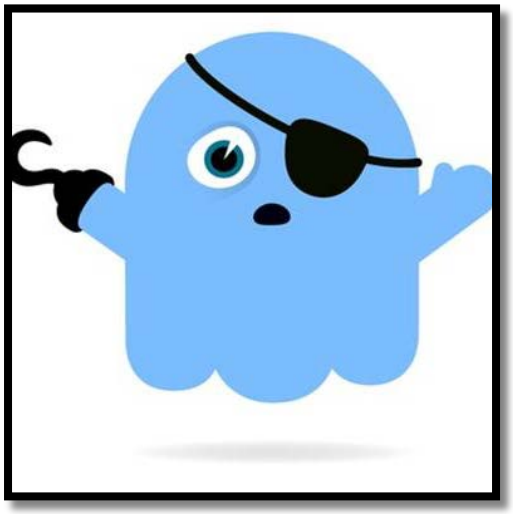
Sample-to-population inference progressions across senior curriculum

***SAMPLE-TO-POPULATION INFERENCE:** The process of drawing conclusions about *population parameters* based on a *sample* taken from the *population*.

Standard	**91035 (1.10): Multivariate data**	**91264 (2.9): Use statistical methods to make an inference**	**91582 (3.10): Use statistical methods to make a formal inference**
CL	Level 5 & Level 6	Level 7	Level 8
Inferential ideas	"Making the call" with L5 or L6 guidelines (based on position of medians and spread)	Using informal confidence interval for population medians (based on sample size & IQR)	Using a formal confidence interval to make an appropriate formal inference (based on the *resampling distribution* of the statistic of interest – *bootstrap confidence interval*)
Key ideas *Clear links between PROBLEM → PLAN → CONCLUSION & the situation should be evident throughout*	• Comparing overall distributions • Understanding of the measures (eg: IQR is a measure of how *spread out* the middle 50% is) • Some appreciation of sampling variability (sample size; if another sample was taken it might look like…)	• Comparing medians • Selecting a *random* sample in an appropriate way • Understanding sampling variability (impact of sample size and population spread)	• Researching context (relating the context-research-findings to what is seen in the data and to any conclusions made – the "so-what?" factor) • Working with the difference between means as well as medians
From Senior Secondary guides	• S6-1: …communicating features in context…	• S7-1: Uses *relevant contextual knowledge* when communicating findings	• S8-1: Uses *informed contextual knowledge* to support explanations and to communicate findings

Use statistical methods to make a formal inference

- Researching context
  - relating the context-research-findings to what is seen in the data and to any conclusions made
  - the “so-what?” factor
- Analysis tool – Bootstrap confidence intervals
  - Working with the difference between means as well as medians



Teaching…*my plan*

- Recap key ideas from Level 7
- Teach new method of analysis (bootstrapping)
- Then drop into PPDAC framework linking to “informed contextual knowledge”

Summary data set

- Fraggle rock clip

http://www.youtube.com/watch?v=l_zsz_MlVvI

Doozers

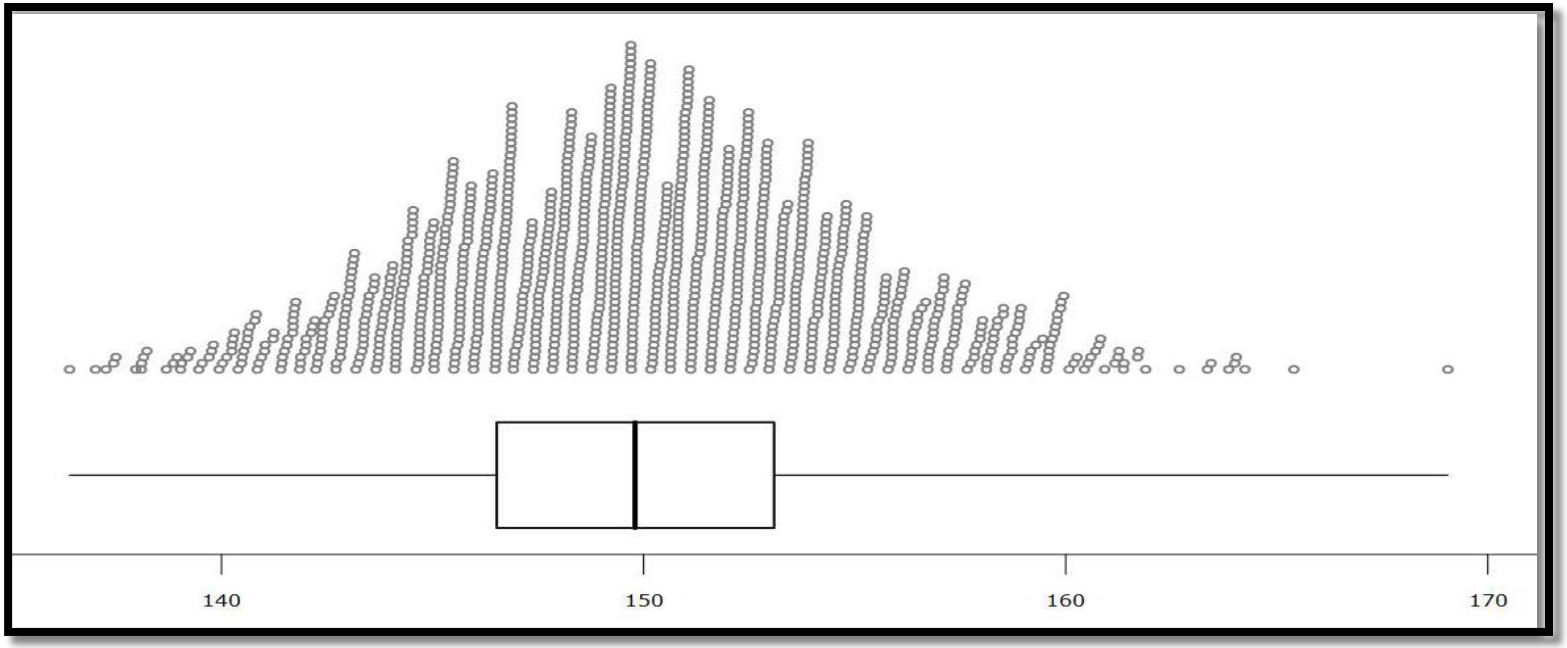
“New” data set to…

- Recap L7 ideas
- Introduce bootstrap distribution and confidence intervals

Doozers

Mean height of Doozers at Fraggle rock is **150mm** (standard deviation of **5mm**)

- Sketch the height distribution of the population of Doozers at Fraggle Rock



Height distribution of the ***population*** of Doozers at Fraggle Rock

Doozers

How tall is **your** doozer?

- Casio Graphics calculator: RandNorm#(5, 150)
- Draw your doozer
- Tell us about your doozer

… in teaching world

- We actually know the population distribution of Doozer heights → we are using this data set as a teaching example.

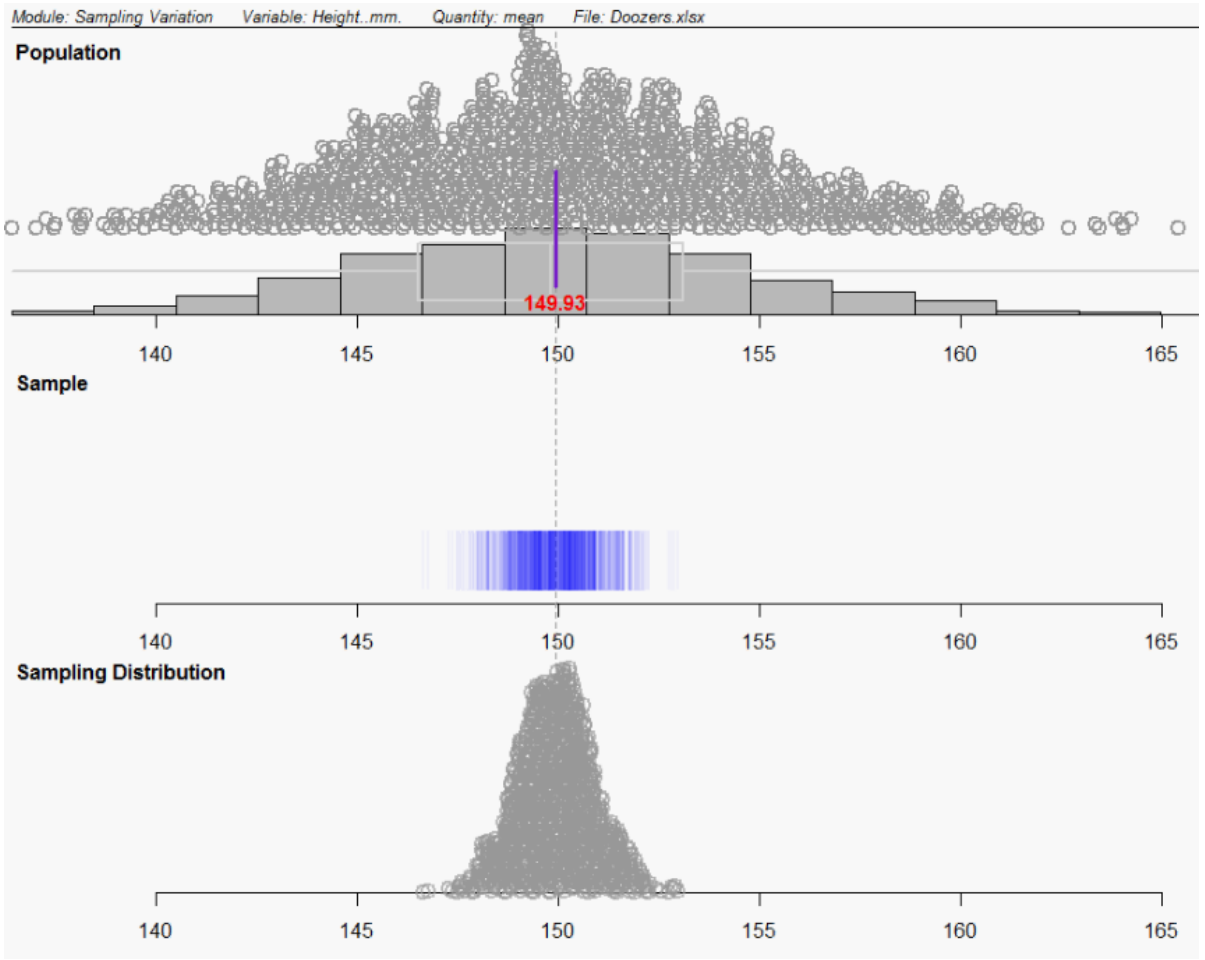
"I wonder what is the mean height of Doozers in Fraggle Rock?"

- How would you have gone about answering this last year?

KEY IDEAS FROM L7

- Random sample
- Informal confidence interval
  - Why we use an interval estimate
- Sampling variation

Sampling distribution



"I wonder what is the mean height of Doozers in Fraggle Rock?"

- Bootstrap method for constructing confidence intervals

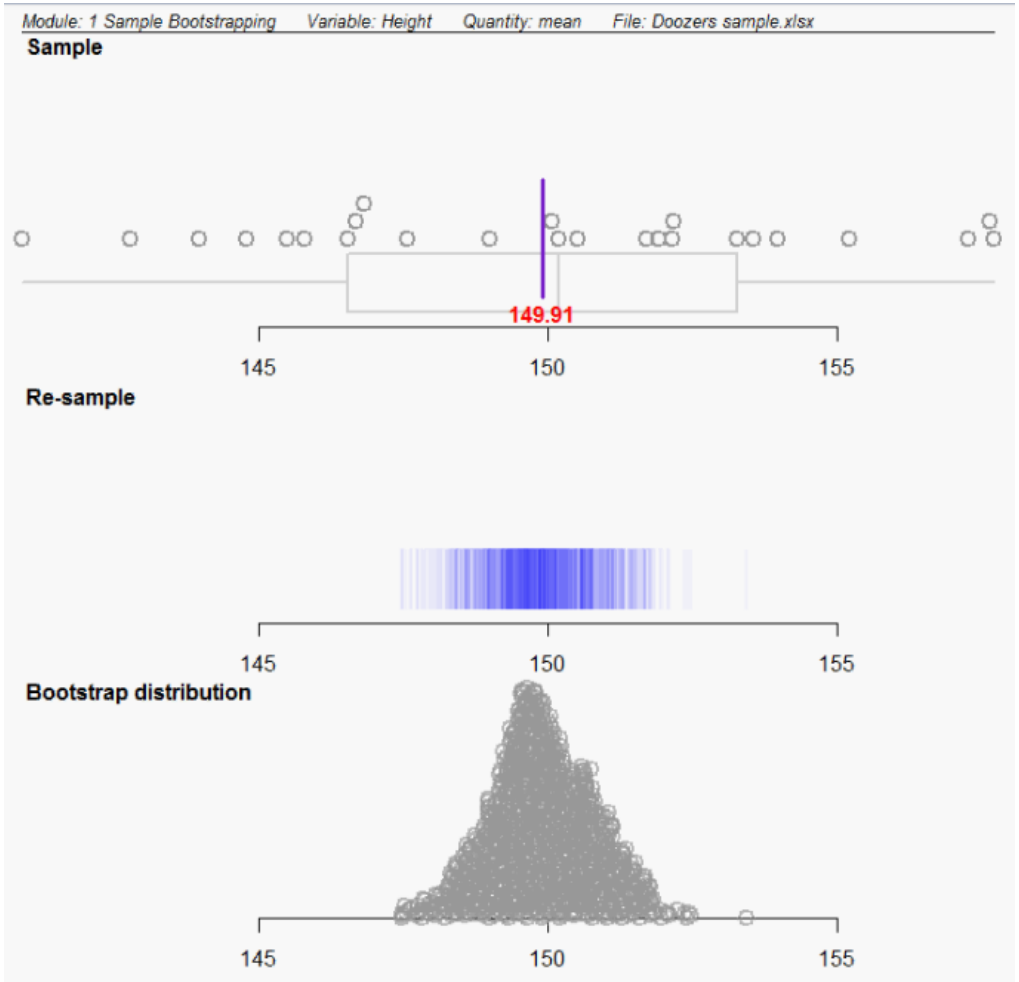
- Random sample of size 25*

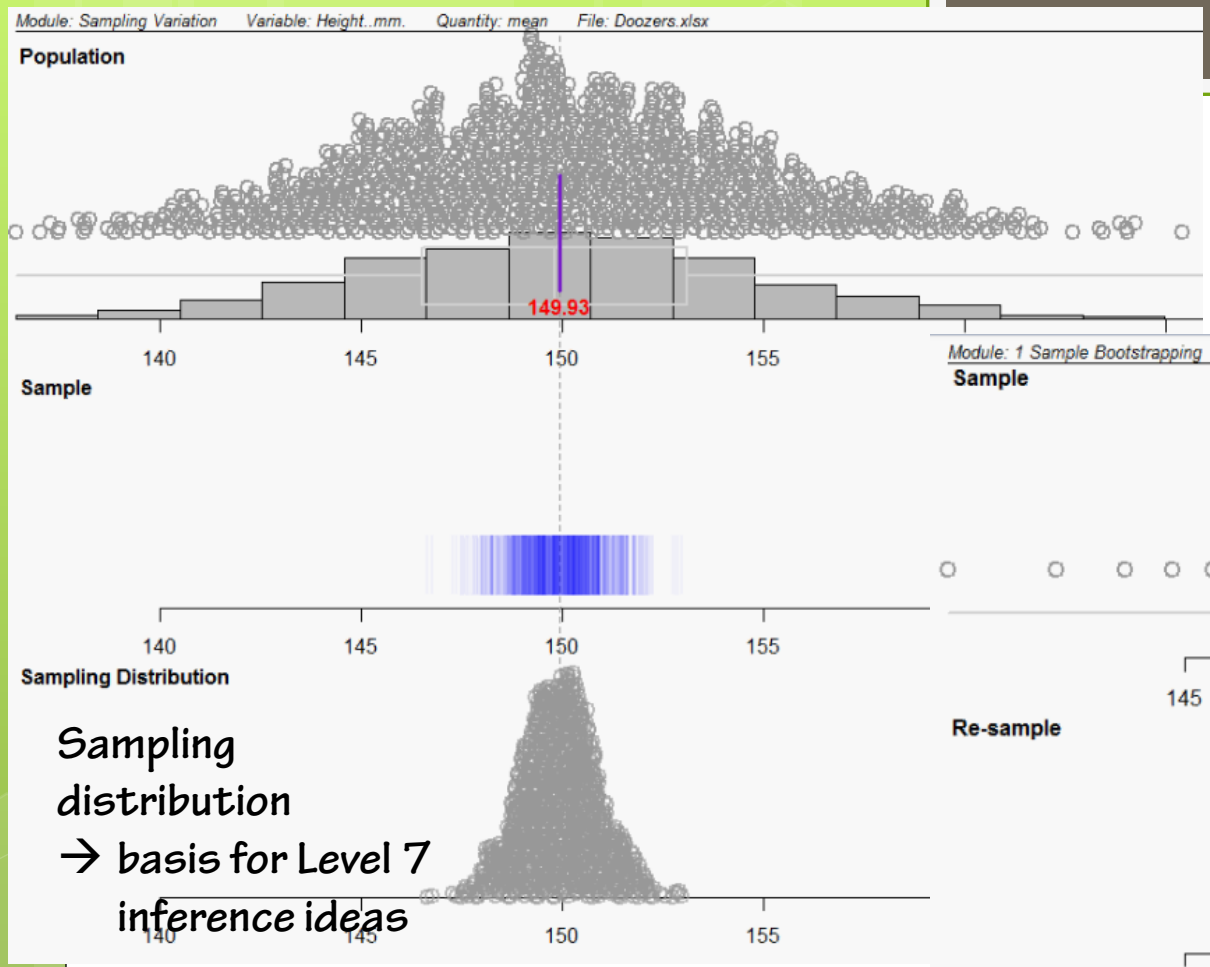
* or appropriate size for your class

Hands on → technology

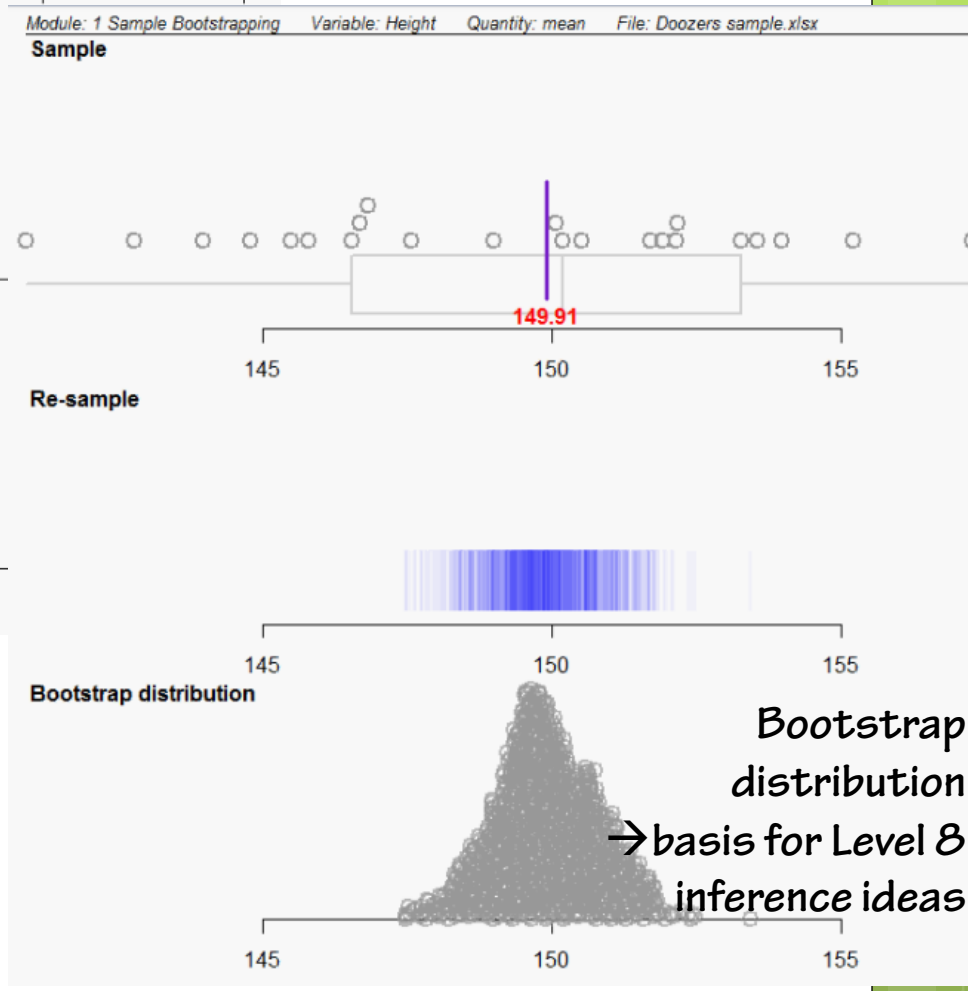
- Re-sample distribution
  - Students complete re-samples individually, then collate class results
- Decide from class results an interval for the population mean
- Complete same re-sampling using iNZight
  - Create bootstrapped re-sampling distribution
  - Confidence interval

Bootstrap distribution





Sampling distribution
→ basis for Level 7 inference ideas

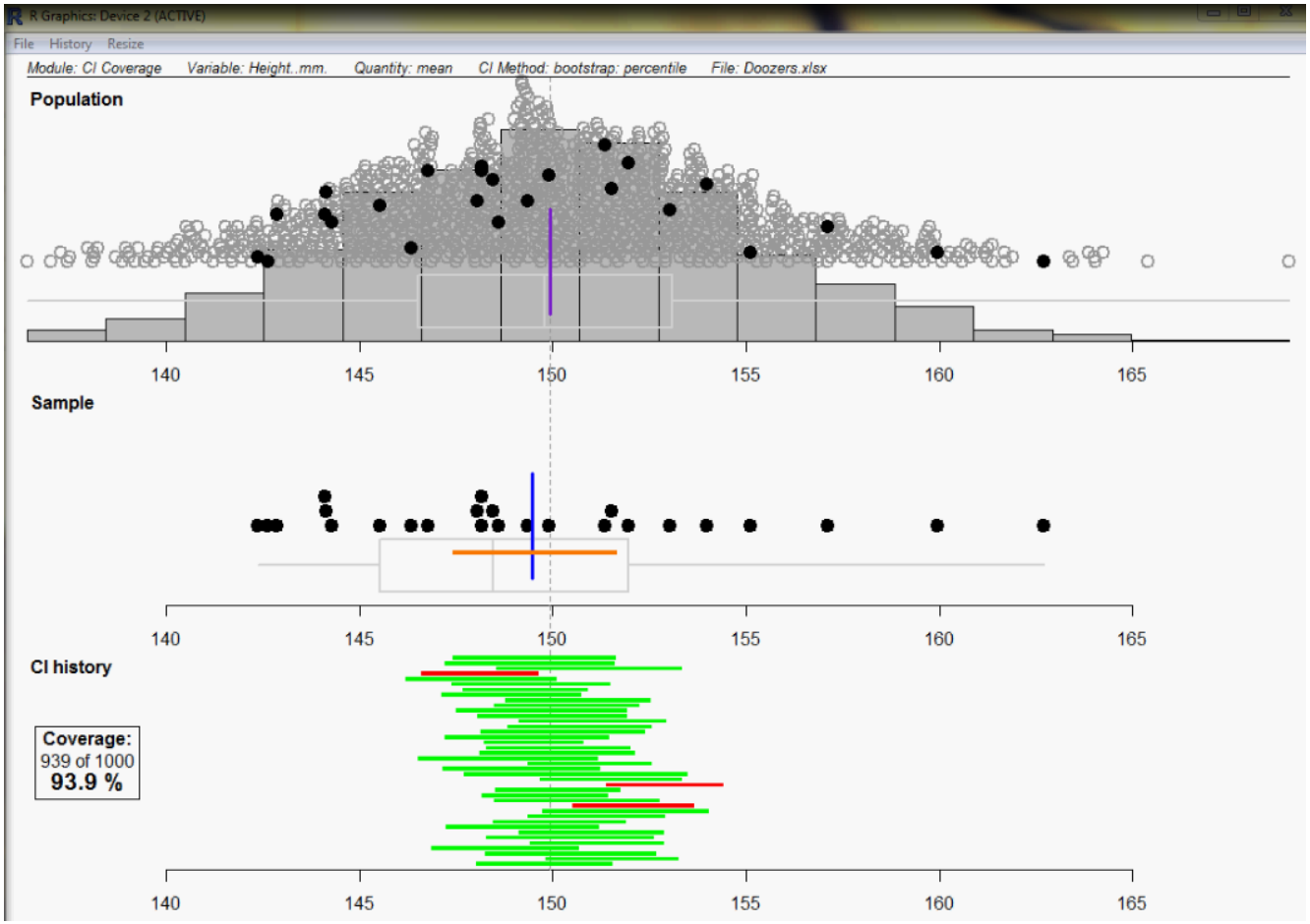


Bootstrap distribution
→basis for Level 8 inference ideas

Method works...

- Check with a known population

Confidence interval coverage



Interpreting CI

- Sample → population link strong
- Some level of uncertainty evident ("pretty sure")
- Population parameter identified
- Context clearly identified

Interpreting CI - activity

- Explain what is wrong with each of these conclusions?

A. From my sample, I am pretty sure that the population height of doozers at fraggle rock is between 148.1mm and 151.6mm

B. From my sample, it's a fairly safe bet that the mean height of doozers is between 148.1mm and 151.6 mm

C. From my sample, I'm reasonably sure that the population mean for doozers at fraggle rock is between 148.1mm and 151.6mm.

The standard

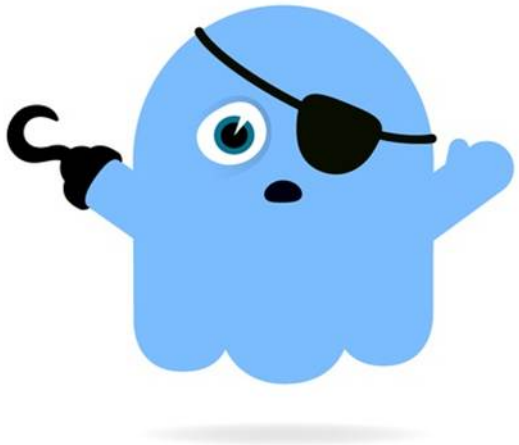
91582 (3.10)

Use Statistical methods to make a formal inference

Use Statistical methods to make a formal inference

- Hook needed to give motivation for context
- EG: SURF dataset → Alasdair Thompson and the “Sorry, but its true” interview

This is to get students to start thinking about the context, and to allow them to go and research into the situation to give purpose to their investigation



Captain Hook

Problem

- Students research into situation to give purpose to investigation and help them choose what variables to investigate
- Clear investigative question
- Prediction of what students expect to see in their analysis and why (based on their research)

Problem

A good comparison question should include

- **Variable that is being examined**
- **Groups that are being compared**
- **Population that inferences are being made about**
- **Statistic that is being estimated**

- *I wonder what the difference in mean heights is between Doozers from Fraggle Rock and Doozers from Raggle Frock.*

Data

- Given multivariate data set (sample)
  - Data set needs to be large enough to allow meaningful analysis
  - Students may choose to re-categorise the data set in different ways from what is provided

Analysis

- Initial examination of sample data including basic graphs and summary statistics
- Bootstrap distribution for the difference in means/medians

Conclusion

- Interpretation of formal confidence interval
  - Sample → population link strong
  - Some level of uncertainty evident ("pretty sure")
  - Population parameter identified

Conclusion

- Correct call, with justification
  - Should reflect investigative question
  - Call based on whether zero is contained within the interval or not
  - Direction of evidence (if zero outside of interval)

Conclusion

- Linking back to the context and using initial research to help explain what this means
  - **the "so what?" factor**

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NZQA Trial paper

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http://nzta.govt.nz/about/advertising/drink-driving/legend.html

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Variable definitions:

Variable	Description
Gender	• Male • Female
Age	Age in years at time of crash
Risk group	Highest risk age group 15 – 24 years (H) Lowest risk age group 50 – 59 years (L)
Licence type	Type of licence held at time of crash • Learners (L) • Restricted (R) • Full (F)
Crash severity	• Minor injury crash (M) • Serious injury crash (S)
Blood Alcohol Level	Recorded blood alcohol level recorded at time of crash *(In milligrams of alcohol per 100 millilitres of blood. Breath test results have been converted to a blood equivalent)*
Distance driven	Estimated distance driven in the last week *(in kilometres)*
Distance from home	Estimated distance away from home when crash occurred *(in kilometres)*
Vehicle age	Age of vehicle involved in crash
Insurance payout	Value of insurance claim
Medical expenses	Estimated medical costs as a result of the crash *(total costs to date)*
Time off work	Estimated number of days off work as a result of the crash (if applicable)

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Problem 	• State the purpose of your investigation. *Remember to explain **why** you are investigating this situation; give a brief background and **hypothesis** to be investigated. You are expected to do background research to understand the situation.* • Identify the variables you wish to investigate. • Pose an appropriate comparison investigative question.
Analysis 	• Select and use appropriate displays and statistical measures. • Discuss and compare sample distributions
Conclusion 	• Make a formal statistical inference. • Conclude your investigation, reflecting on your hypothesis and justifying your formal inference *This may include:* – *Discussing sampling variability, including the variability of estimates.* – *Reflecting on the process you have used to make the formal inference*

Remember: The quality of thinking demonstrated in your report and your ability to link the context and populations to the different components of the statistical enquiry cycle will determine your overall grade. As you write your report, take care to link your discussion to the context and support your statements by referring to statistical evidence.

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Mathematics and Statistics 91582 (3.10) version 1:
Use statistical methods to make a formal inference **Credits:** 4

Final grades will be decided using professional judgement based on a holistic examination of the evidence provided against the criteria in the AS.

	Achieve Use statistical methods to make a formal inference	**Merit** Use statistical methods to make a formal inference, with justification	**Excellence** Use statistical methods to make a formal inference, with statistical insight
	The student shows evidence of using each component of the statistical enquiry cycle to make an inference Examples of using each component of the statistical enquiry cycle appear below	The student makes an inference, showing evidence of linking components of the statistical enquiry cycle to the context, and/or populations and referring to evidence in support of statements made. Examples of linking components of the statistical enquiry cycle to the context and/or populations and referring to evidence such as sample statistics, data values, or features of visual displays in support of statements made appear below	The student makes an inference with statistical insight, showing evidence of integrating statistical and contextual knowledge throughout the statistical enquiry cycle. They may reflect on the process or consider other explanations. Examples of integrating statistical and contextual knowledge throughout the statistical enquiry cycle appear below
Judgement	***Judgement**: The report covers all the points below to the depth indicated by examples. Minor errors or omissions should not withhold awarding of the grade.*	***Judgement**: The report covers all the points below to the depth indicated by examples. Understanding of key concepts (difference between sample statistics and population parameters; sampling variability; interpretation of the bootstrap confidence interval) should be evident. Minor errors or omissions should not withhold awarding of the grade.*	***Judgement**: The report covers all the points below to the depth indicated by examples. Understanding of key concepts (difference between sample statistics and population parameters; sampling variability, interpretation of the bootstrap confidence interval) should be evident. The student has reflected on the process or has given explanations by considering, in context, the effect of aspects such as sample representativeness on the estimate. They have discussed aspects of the investigation in context, such as re-examining the data to show a different aspect, or justify a point being made. Minor errors or omissions should not withhold awarding of the grade.*
Problem	Posing a comparison investigation question using a given multivariate data that clearly identifies the comparison and the population(s) *Question should be about the difference in medians or means*	Posing a comparison investigation question using a given multivariate data set including giving an explanation for the choice of variables for their investigation to pose an appropriate question and a hypothesis about what they may find *Question should be about the difference in medians or means*	Posing a comparison investigation question using a given multivariate data set including using research to develop the purpose for their investigation and using this contextual knowledge to pose a question. The purpose should include a hypothesis and justification of the hypothesis *Question should be about the difference in medians or means*

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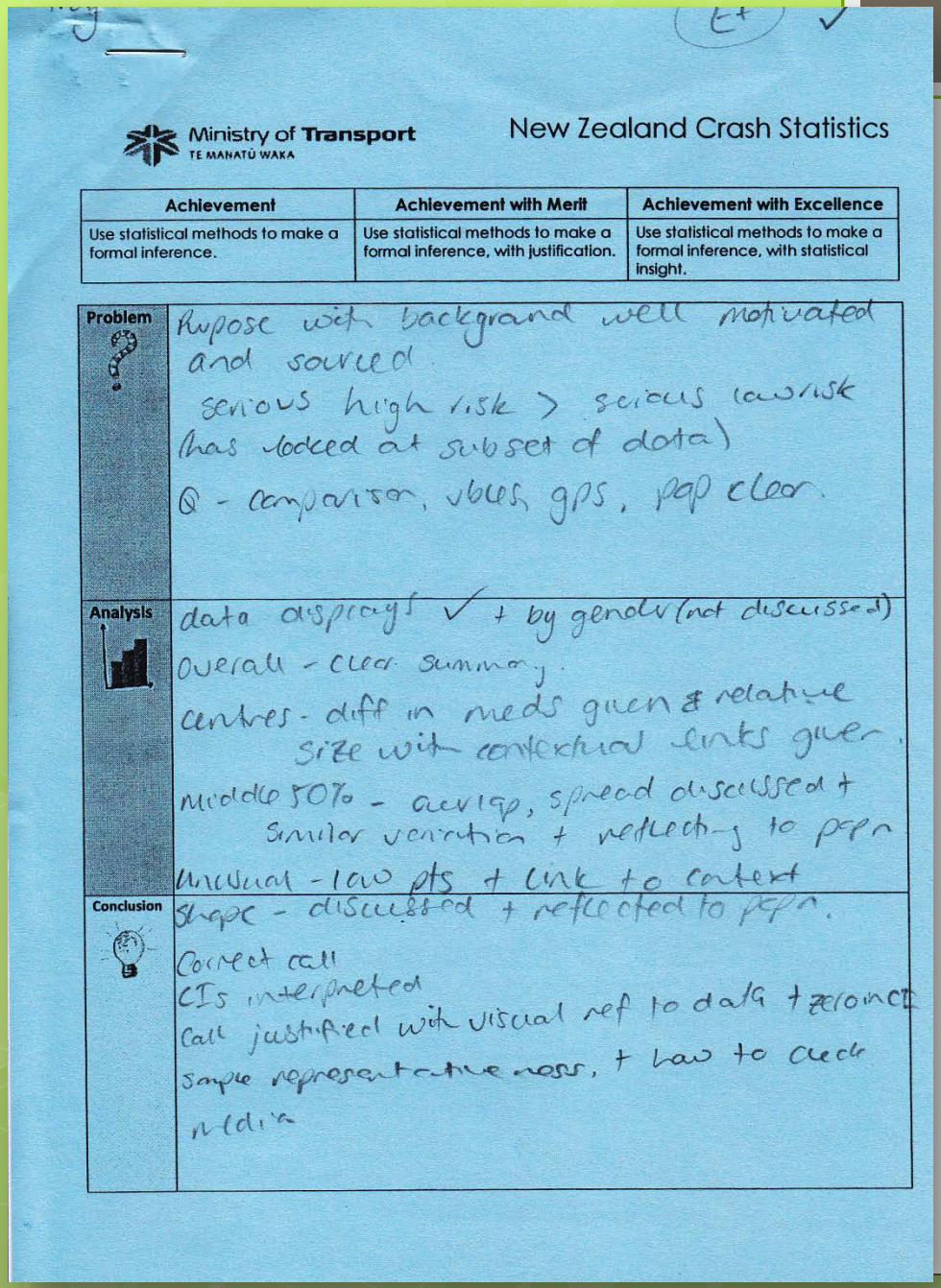
Analysis	Selecting and using appropriate displays and summary statistics. This includes producing dot plots, box plots, summary statistics including difference between sample medians or means and constructing a bootstrapped confidence interval Discussing and comparing sample distributions by identifying and comparing features of them. This could involve comparing the shift/centre, spread, shape and unusual features, using features of the displays and the summary statistics.	Selecting and using appropriate displays and summary statistics. This includes producing dot plots, box plots, summary statistics including difference between sample medians or means and constructing a bootstrapped confidence interval Discussing and comparing sample distributions by identifying and comparing features of them. This will involve comparing the shift/centre, spread, shape and unusual features with reference to features of displays and summary statistics **AND** links to context or investigative question.	Selecting and using appropriate displays and summary statistics. This includes producing dot plots, box plots, summary statistics including difference between sample medians or means and constructing a bootstrapped confidence interval Discussing and comparing sample distributions by seeking explanations for features of the data identified in displays or summary statistics and considering the impact of these on the context or investigative question.
Conclusion	Making an appropriate formal statistical inference: using the resampling (bootstrapping) method to construct a confidence interval. Communicating findings clearly in a conclusion using the formal inference to answer the investigative question.	Making an appropriate formal statistical inference. For example: using the resampling (bootstrapping) method to construct a confidence interval. Communicating findings clearly in a conclusion using the formal inference to answer the investigative question, justifying the call and making links to the context.	Making an appropriate formal statistical inference. For example: using the resampling (bootstrapping) method to construct a confidence interval. Communicating findings in a conclusion using the formal inference to answer the investigative question in context. This includes justifying the call **AND** linking back to the purpose of the investigation; considering other explanations for the findings; reflecting on the process (which may include discussion on sampling variability).

NOTES:

- Students are given a 'motivation' into the situation, but not too much of a direction in terms of hypothesis. It is expected that they then research themselves and develop their hypothesis and investigative question
- This research into the context will allow them to reflect on what they see with further depth
- There should be an obvious connection between a student's question, analysis and conclusion; they should have answered their question
- There should be population/sample understanding evident at all levels
- Understanding of sampling variability is implied through use of a confidence interval for population parameter.



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Achievement	Achievement with Merit	Achievement with Excellence
Use statistical methods to make a formal inference.	Use statistical methods to make a formal inference, with justification.	Use statistical methods to make a formal inference, with statistical insight.

Problem

Rupose with background well motivated and sourced
serious high risk > serious low risk
(has looked at subset of data)
Q - comparison, vbles, gps, pop clear.

Analysis

data displays ✓ + by gender (not discussed)
Overall - clear summary.
Centres - diff in meds given & relative size with contextual links given.
Middle 50% - overlap, spread discussed + similar variation + reflecting to pop^n
Unusual - low pts + link to context

Conclusion

Shape - discussed + reflected to pop^n.
Correct call
CIs interpreted
Call justified with visual ref to data + context
Sample representativeness, + how to check
media

Marking grid…

Student work

High not achieved - Becky

"The confidence interval shows that drivers of the higher risk age group tend to have a higher blood alcohol level at the time of crash, by somewhere between 0.0 and 24.0 milligrams of alcohol per 100 millilitres of blood."

Pg 5, Analysis, *end of* 1st paragraph

CI interpretation

- **Population unclear**
- **Population parameter not mentioned**

Low level of analysis comments along with incorrect means HIGH NOT ACHIEVE *rather than* LOW ACHIEVE.

High achieved - Josh

"I would like to look at driver blood alcohol level and the severity of the crash. I wonder what the difference in median blood alcohol levels and severity of a car accident"

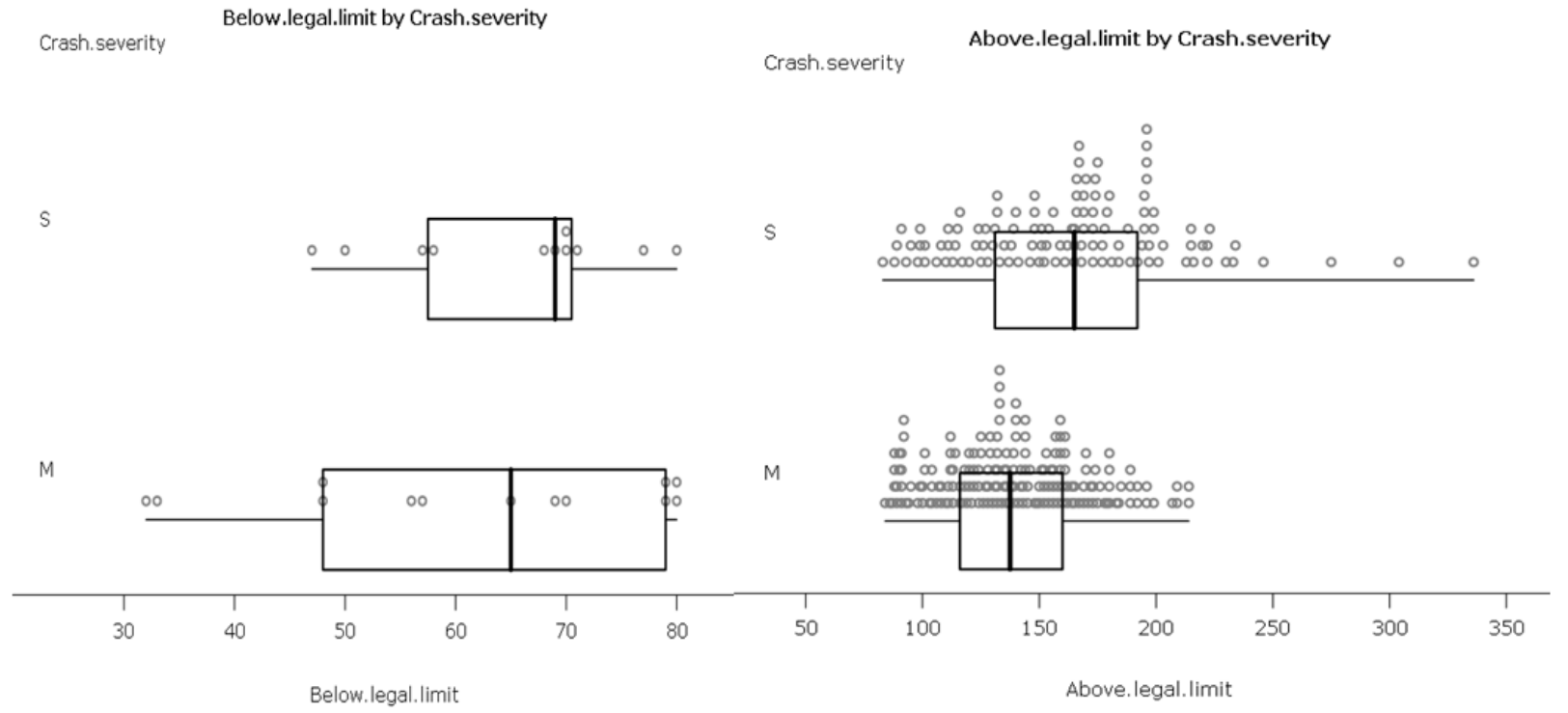
Very top of Page 7

Problem

- Question about difference in medians
- Groups being compared not well defined
- Population not given

Sample → population link unclear in investigative question and CI interpretation; starting to make contextual links and thinking about the "so what" but comments not strong hence HIGH ACHIEVE *rather than* LOW MERIT

High merit - Logan



Split of data a nice idea, but didn't really contribute much further depth to the analysis. Contextual links not strong, especially in analysis hence HIGH MERIT *rather than* LOW EXCELLENCE

High merit - Logan

"…Although I think that in the population there would be more points of higher blood alcohol levels. The few points from this sample that we have of under the blood alcohol limit show a relatively even spread which I think is reasonable for the population because these crashes may or may not have been caused by the alcohol in the persons system."

End first paragraph, page 10

Analysis

- … data re-categorised by legal BAL limit and described with link to population

Split of data a nice idea, but didn't really contribute much further depth to the analysis. Contextual links not strong, especially in analysis hence HIGH MERIT *rather than* LOW EXCELLENCE

Low excellence - Alice

"…I think this because the amount of advertisements we see in the media to do with stopping people from drink driving must mean that there is a high level of danger involved with drinking and driving. Otherwise the government would not spend money on trying to advertise this fact."

Third paragraph, page 11

Problem

- … justification of hypothesis opinion rather than referenced

Split of data a nice idea, but didn't really contribute much further depth to the analysis. Contextual links show depth of thinking, but some personal interpretation of information rather than based on referenced material hence LOW EXCELLENCE *rather than* HIGH EXCELLENCE.

Low excellence - Alice

"…This makes sense because it is possible to have a serious injury crash even without the influence of strong intoxication. Which is why the data for the serious injury crashes BAL's vary more."

Middle 50% - Page 12

Analysis

- … good contextual links and thinking evident about population but based on personal opinion rather than factually referenced

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Low excellence - Alice

"…I believe that this makes sense in the context because in the minor injury crashes although there was alcohol in their blood that is not necessarily what caused the crash, and people with this much alcohol in their blood probably do get away with driving a lot of the time without a crash. It also makes sense in the case of the serious injury crashes being right skewed because not many people would consider driving with as much alcohol in their systems as the few outliers that did drive, resulting in a crash."

Shape - Page 12

Analysis

- … good contextual links and thinking evident about population but based on personal opinion rather than factually referenced

Low excellence - Alice

"One thing I am interested and surprised about though is the fact that as seeming by these results you get to a certain point when you are so intoxicated you are almost definitely going to have a serious injury crash. Once your BAL gets past about 220 it looks like if you crash you will almost definitely come under the serious injury crash category. This is interesting and I hope will convince some of the students I have done this investigation for to not, in the future, get drunk before driving."

Last paragraph, page 12

Conclusion

- … comments relating findings back to context

Split of data a nice idea, but didn't really contribute much further depth to the analysis. Contextual links show depth of thinking, but some personal interpretation of information rather than based on referenced material hence LOW EXCELLENCE *rather than* HIGH EXCELLENCE.

Troy

E+ ✓

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Call justified with visual ref to data + zero in CI
Sample representativeness, + bias to check
media

High excellence - Troy

My best paper ☺

Mathematics & Statistics Faculty

Michelle Dalrymple

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