STATISTICAL LITERACY: FACTUAL ASSESSMENT TO SUPPORT HYPOTHETICAL THINKING

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The GAISE College report suggested that teachers assess statistical literacy by students "interpreting or critiquing articles in the news." Media stories typically present summary statistics to support non-statistical conclusions. Summary statistics require hypothetical thinking which in turn requires drill in factual exercises involving deductive right-wrong answers. This paper presents a wide range of deductive exercises that may help students develop the hypothetical thinking needed to deal with the fact that all statistics are socially constructed. This paper presents 130 different topics involving fact-based exercises with objective answers. Of these, 50% are numerical, 30% are number-related and 20% are non-numeric. Selected examples are presented. At least half of these exercises have been used by students in a web-based format. These exercises are classified by topics in traditional research statistics and in statistical literacy.

ASSESSING STATISTICAL LITERACY

The design and assessment of a course depends critically on the goals of the course and on the background and interests of the students. A statistical literacy course has different goals and types of students from a traditional statistics course. Therefore the assessment exercises and activities will differ from those in a traditional statistics course.

The ASA recently endorsed Guidelines for Assessment and Instruction in Statistics Education (GAISE, 2005). The GAISE College report recommended that introductory courses in statistics should strive to emphasize statistical literacy, stress conceptual understanding and integrate assessments that are aligned with course goals to improve as well as evaluate student learning. This report defined statistical literacy as "understanding the basic language of statistics (e.g., knowing what statistical terms and symbols mean and being able to read statistical graphs), and understanding some fundamental ideas of statistics."

There are many choices for which ideas are fundamental. Moore (2001) distinguished statistical literacy ("What every educated person should know about statistical thinking") from statistical competence ("roughly the content of a first course for those who must deal with data in their work … or what we hope a statistics student will retain five years later"). Utts (2003), Schield (2004a, 2004b) and Moreno (2005) have each identified different statistical topics they believed would be necessary to analyze newspaper articles, to make personal health inquiries and decisions, and to understand polls, political, and advertising claims, i.e., to become better decision-makers. But Gal (2002, 2003) noted, "no comparative analysis has so far systematically mapped the types and relative prevalence of statistical and probabilistic concepts and topics across the full range of statistically-related messages or situations that adults may encounter and have to manage in any particular society. Hence, no consensus exists on a basis for determining the statistical demands of common media-based messages." Statistical literacy is still in its infancy.

Best (2001, 2002) argued that regardless of what particular statistical concepts are used in the everyday media, "all statistics are socially constructed" – defined, selected, measured, compared and presented by people with choices and motives. Schield (2007) noted "the less data available, the less that can be known about the effects of social construction. Media stories typically present only a few carefully-selected summary statistics so the influence of social construction on these statistics cannot be seen in the data presented. In such cases, readers must be most careful in drawing conclusions from such summaries."

HYPOTHETICAL THINKING

Schield (2007) noted that "*This lack of access to the underlying data requires hypothetical thinking in order to analyze or evaluate essays that use statistics as evidence. This hypothetical thinking is absolutely critical once one accepts that all statistics are socially constructed – they are not numerical absolutes: they are selected, defined and presented by people who have motives in seeing the statistics be large or small.*"

This social-construction-of-statistics idea is extremely important when the only data given are selected summary statistics. Readers can't compare mean with median, we can't determine the influence of an outlier. Readers can't determine how a different definition would influence the size of a statistic. In each case analyzing and evaluating the size of a statistic requires hypothetical thinking: thinking about alternate ways in which statistics could have been defined, collected, formed and presented.

FACTUAL THINKING

Students have difficulty thinking hypothetically or inductively. They are used to looking for clues to the answer inside the problem or data. They aren't used to thinking outside the 'box.' They need factual (deductive right-wrong) exercises to develop their skill. This relationship is shown in Figure 1.

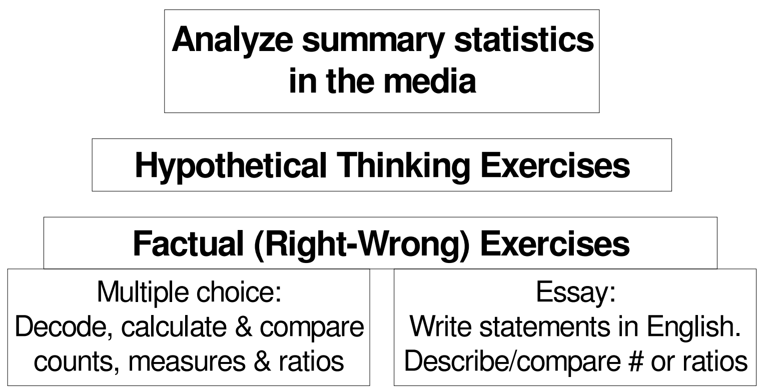

Figure 1: Statistical Literacy Assessment Pyramid

Students need to see how changing the definition of a group or activity can influence the count or the measurement. They must see how taking into account a related factor can influence the size of an association.

This paper presents a wide range of factual statistical literacy exercises (bottom boxes). Hopefully these exercises will help students' ability to think hypothetically (middle box) so they can better analyze and evaluate media essays (top box). Mastering these exercises is a key element in being statistically literate. But until we know the prevalence of various statistics in the everyday media, the emphasis one should give to different exercises is unknown.

Some statistical topics (such as Simpson's paradox) have been de-emphasized historically because they did not lend themselves to problems and exercises. Some of these exercises present new ways of teaching such topics. Studying these exercises may encourage statistical educators to rethink their choice of topics.

Mathematically, some of these activities may seem too elementary. But the primary goal is not to introduce the students to higher-level mathematics or even to help students obtain a detailed understanding of a mathematical concept (e.g., standard deviation or correlation). In statistical literacy, the primary goal of factual exercises is to help students develop a facility for hypothetical thinking about summary statistics presented in the everyday media.

Appendix A lists 130 different types of exercises involved in the W. M. Keck Statistical Literacy project. It is unlikely that anyone teaching statistical literacy would cover all these topics or exercises. The purpose of this list is to present a comprehensive range of exercises to address the needs of teachers having different approaches to teaching statistical literacy. The mathematics involved in these exercises is indicated in the formulae in Appendix B.

EXAMPLES OF FACTUAL EXERCISES

Here are examples of objective right-wrong exercises. They were chosen because they are not typically included in a traditional introductory statistics course.

1. *What percentage of the white-black income gap is attributable to family structure?* Schield (2006). Exercise C3I.
2. *Describe a percentage or compare two percentages in a table or graph.* Is "Widows are more likely among suicides than widowers" the same as "Widows are more likely to commit suicide than widowers"? Burnham and Schield (2005). Exercises C42 and C52.
3. *Calculate the percentage of cases attributable to a treatment or exposure.* In the US in 2002, the percentage of newborns which have low birth-weight is approximately 12% among mothers who smoke (8% among mothers who don't smoke). Among mothers who smoke, what percentage of low-weight births are attributable to the mother smoking? US Statistical Abstract 2006 Table 86. Exercises C5A-B
4. *Calculate the number of cases attributable to a treatment or exposure.* In the US in 2003, the poverty rate was 25% in single-parent homes (5% in married-family homes). There are 4.5 million single-parent homes. How many of the single-parent families in poverty are attributable to being headed by a single-parent? US Statistical Abstract Table 699. Exercise C5C.
5. *Calculate an inverse percentage using related data.* Suppose that 72% of those in prison did not graduate from high school whereas 12% of those 25-35 did not graduate from high school. If 5% of all high school students end up going to prison, what is the chance that a high school student who fails to graduate from high school will end up in prison? US Statistical Abstract 2006, table 217. For more on Bayes comparisons, see Schield (2004b).
6. *Determine which of two related three-factor percentages is greater.* Which is bigger, "the percentage of infant deaths which are due to birth defects" or "the percentage of infants who die due to birth defects"? Which is bigger, P(A|BC) or P(AB|C)? Schield (2005). *Exercise C6B.*
7. *What percentage of the difference in hospital death rates is attributable to patient condition?* Schield (2004a). Exercise C6J.
8. *How many times one would need to flip a set of 10 coins so it is more likely than not that at least one of these sets will come up all heads.* Schield (2005). Ex. C7E.
9. *Calculate the influence of third factor on the size, direction and statistical significance of an association.* Schield (2004c). Exercises C7N-P.

These sample questions illustrate some of the differences between the exercises in traditional statistics and those in statistical literacy. For more examples see Schield (2007).

CLASSIFICATION OF FACTUAL EXERCISES

There isn't room in this paper to present examples of all 130 exercises shown in Appendix A. But these exercises can be classified to see how they compare with those in other courses. Exercises can be classified based on whether the right-wrong exercise involves a multiple-choice format or whether the activity involves writing a single statement that can be machine-assessed as right or wrong. These are the two blocks in the bottom row of

Figure 1. For a discussion on the online program used to evaluate student writing, see Burnham and Schield (2005).

A second way classifies exercises by their mathematical nature: do they have a number as a result (1), do they describe, compare or communicate a numeric relationship (2), or are they non-mathematical (3) – they involve broader critical thinking issues such the distinction between association and causation.

Table 1 classifies these 130 types of exercises by these two indexes. While most (91%) of the right-wrong exercises are multiple choice, the 9% that are single-sentence statements using ordinary English are extremely important in communicating mathematical concepts accurately and succinctly. See Schield and Burnham (2005).

Table 1:
Exercises by Multiple Choice and Type of Math

Multiple choice	**Mathematics**				
	(1)	(2)	(3)	Total	
No (0)	5	7		12	9%
Yes (1)	60	38	20	118	91%
TOTAL	65	45	20	130	100%
	50%	35%	15%	100%	

STATISTICAL INFERENCE CLASSIFICATION

Exercises can be classified by their statistical content: critical thinking (T), descriptive statistics (D), comparison of numbers (C), conditional probability (P), the comparison of probabilities (L), standardization (S) and randomness/inference (R). Table 2 classifies the 130 exercises by the type of statistics (rows) and the type of mathematics (columns).

Table 2:
Exercises by Type of Statistics and Type of Math

Traditional Statistics	**Stat**	**Mathematics** 1	**2**	**3**	**TOTAL**	
Critical thinking	T		2	16	18	14%
Descriptive	D	16	3	4	23	18%
Compare #	C	2	5		7	5%
Rates %	P	6	17		23	18%
Likely compare	L	5	16		21	16%
Standardizing	S	20	2		22	17%
Randomness	R	16			16	12%
TOTAL		65	45	20	130	
		50%	35%	15%	100%	

Mathematics content: Type 1 math problems (50%) have numerical answers, Type 2 (35%) communicate mathematical relationships and Type 3 (15%) involve things that are not mathematical (e.g., association versus causation).

Traditional statistics content: Critical thinking (14%) includes topics such as causation and association. *Descriptive statistics* (18%) and *randomness/inference* (12%) are self-defining. The remaining types of exercises may need more explanation.

Comparison of numbers (5%) involves the math and grammar needed to calculate and communicate the various types of arithmetic comparisons: difference, ratio and percentage different/change. E.G., 8 is 4 times as much as 2, but 3 times (300%) more than 2. Schield (2004b).

Rates/percentages (18%) involve conditional probability. In a traditional statistics text, these involve union, intersection, and independence. In statistical literacy, conditional probability involves calculating, reading/decoding, writing and interpreting part-whole relationships between groups of subjects and their conditions or activities.

- Calculating involves calculating a percentage from tables of counts.
- Reading/decoding involves identifying the part and whole in ordinary English statements and in questions that use different ratio grammars. Do these two questions ask the same thing? "What percentage of men are smokers?" versus "What is the percentage of men who are smokers?" Do these two statements mean the same thing? "The percentage of women who are runners" versus "The percentage of women

among runners." This activity may involve reading rates and percentages as presented in tables and graphs. See Schield (2004b) for more detail.

- Writing involves using ordinary English to describe a single ratio (rate or percentage) or compare two ratios when presented in a table of rates or percentages, or based on a table of counts. Students find that writing is much harder than reading or decoding. See Burnham and Schield (2005) for a discussion of a web-based program that decodes the semantics from ordinary English syntax.
- Interpreting involves a number of distinct activities. One exercise involves identifying which of two ratios having the same terms is larger or smaller. E.g., "The percentage of male smokers who are runners" or "The percentage of smokers who are male runners." "Among live births, the percentage who died due to birth defects" vs. "the percentage of infant deaths which are due to birth defects."

Likely Comparisons or Comparing probabilities (16%) also involves merging the comparison of numbers with the description of ratios to compare ratios (rates and percentages). In statistical literacy, the comparison of probabilities involves the same activities as in conditional probability: calculating, reading/decoding, writing and interpreting. See Schield (2004b).

- Calculating involves calculating a comparison of two ratios from two descriptive statements involving rates or percentages, from a table or graph of counts, or from a table or graph of percentages or rates.
- Reading/decoding involves identifying both the part and whole and the test and base in statements such as "Accidental deaths are more likely for men than for women" or in "Men are more likely among people that die accidentally than are women."
- Writing involves using ordinary English to compare two ratios. Students should be able to translate rates between clause grammar ("Men die accidentally at a higher rate per year than women") and phrase grammar ("The rate of accidental deaths per year is higher for men than for women").
- Interpreting involves many activities. One activity involves assessing whether a percentage is backward: E.g., "Most accidents occur within 25 miles of home." Does this mean "accidents are more likely to occur near home than further away"? Not necessarily. A second activity involves over-involvement ratios. If the readers of Thomas Paine's "Common Sense" were more likely among those favoring separation from Great Britain than among the general population, then we can deductively conclude that readers of "Common Sense" were more likely to support separation from Britain than were those in the general population. A third activity involves a Bayes comparison. If 72% of prison inmates didn't graduate from high school and if 12% of young adults didn't graduate from high school, then we can say that those high-school age students who don't graduate are six times as likely to go to prison as are those in the general population. So if 1% of the population goes to prison, then we expect that 6% of high-school students who don't graduate will go to prison.

Standardization (17%) is an essential topic in statistical literacy. Standardization includes several techniques that take into account the influence of a related factor. In this paper, comparisons, averages and ratios (percentages and rates) are treated as separate topics so standardization includes

- Simple scaling: Z-scores and their normalizing to new scales, the coefficient of variation (standard deviation scaled by the mean), effect size (the difference between two means scaled by their pooled standard deviation) are all examples of scaling by a related factor.
- Adjusted weighted averages: Calculating the change in the weighted average that occurs when the mixtures (the size of the subgroups) are made the same. This adjustment can be done graphically when the confounder is a binary variable. Since the confounder is binary, there is less need for diagnostics and checking of model as-

sumptions. This activity introduces multivariate thinking: a key idea in statistical literacy. See Schield (2006).

- Statistical significance of adjusted weighted averages: The penultimate use of standardization is to analyze the influence of a confounder on the statistical significance of the difference between two sample means.

STATISTICAL LITERACY CLASSIFICATION

A fourth way classifies exercises by the type of influence involved: Randomness, Error/bias, Context/confounding and Assembly. The first two categories are quite well known. Randomness includes chance, margin of error, confidence and statistical significance. Error or bias includes subject bias, measurement bias and sampling bias.

The last two categories (Context and Assembly) may be less familiar. Context or confounding involves the influence of factors that are related to the association of interest and were not taken into account by the study design. Assembly involves the choices made in defining groups, in the choice of statistics and their presentation. In both cases, the issue is hypothetical. What could have been done that was not done? This hypothetical thinking is very different and difficult for students that are accustomed to analyzing what is given in a problem or a case.

Table 3 illustrates the distribution of statistical literacy exercises by statistical literacy categories (rows) and by type of mathematics (columns).

Table 3:
Exercises by Type of Statistical Literacy Influences and Math Categories

Statistical Literacy	**Care**	**Mathematics 1**	**2**	**3**	**TOTAL**	
Critical thinking	T		2	16	18	14%
Confounding	C	23	34		57	44%
Assembly	A	18	6		24	18%
Randomness	R	23	1	2	26	20%
Error/bias	E	1	2	2	5	4%
TOTAL		65	45	20	130	100%
		50%	35%	15%	100%	

Table 4 illustrates the distribution of the statistical literacy exercises by the traditional statistics categories (rows) and by the aforementioned statistical literacy categories (columns).

Table 4:
Exercises by Type of Statistics and StatLit Categories

Traditional Inferential Statistics	**Stat**	**Statistical Literacy (CARE)** **T** **Crit.Think**	**C** **Context**	**A** **Assembly**	**R** **Randomness**	**E** **Error/bias**	**TOTAL**	
Critical thinking	T	18					18	14%
Descriptive	D		1	12	5	5	23	18%
Compare #	C		2	5			7	5%
Rates %	P		23				23	18%
Likely compare	L		15	6			21	16%
Standardizing	S		16	1	5		22	17%
Randomness	R				16		16	12%
TOTAL		18	57	24	26	5	130	100%
		14%	44%	18%	20%	4%	100%	

Once again we see a difference between traditional inferential statistics and statistical literacy. Standardization taken broadly involves "taking something related into account." This includes comparing numbers (C=5%), conditional probability (P=18%), comparing probabilities (L=16%) and standardizing for the influence of a binary confounder (S=17%). As such, standardizing taken broadly (56%) includes more than four times as many more different kinds of exercises as does randomness (R=12%).

CONCLUSION

In terms of the traditional categories of statistical education, the most obvious features of these statistical literacy activities is the small percentage that involve just randomness and inference (12%) and the large percentage that involve "taking into account" (56%). In a traditional introductory statistics course these two percentages might be reversed. When viewed from the five categories used for Statistical Literacy, context/confounding (44%) is seen as the dominant theme. Based on this allocation of activities, statistical literacy focuses much more conditional probability and confounding than does traditional statistics.

Much more work will be required to assess how well these factual exercises help students understand key concepts in statistical literacy (such as assembly and confounding), and think hypothetically about the influences on a selected summary statistic in the media.

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APPENDIX A:

Listed are 130 types of right-wrong exercises in the W. M. Keck Statistical Literacy Project. At this point, about half of these exercises have been field-tested by students in a web-based (Moodle) environment. For each exercise, there are five indicators.

The ID number identifies the chapter (2nd character) in the Schield Statistical Literacy textbook while the third character distinguishes the exercises within that chapter.

The MC indicator (MC) indicates whether the exercise is multiple choice (1) or a right-wrong writing activity (0).

The Math indicator (MTH) indicates whether the activity is numeric (1), involves a numerical relationship (2) or is non-numeric (3).

The StatLit indicator (CARE) indicates whether the exercise involves critical thinking (T) or the influence of Context/confounding (C), Assembly (A), Randomness (R), or Error/bias (E).

The Stat indicator (STAT) indicates whether the activity involves critical thinking (T), traditional descriptive statistics (D), randomness/inference (R), comparison of numbers (C), conditional probability (P for percent/percentage), the comparison of probabilities (L for likely), or standardization (S).

The 29 exercises in Chapters 1 and 2 deal with the foundations of statistical literacy.

ID	MC	Mth	care	stat	**CHAPTER 1:**	
C1A	1	3	T	T	Distinguish observable from unobservable	14
C1B	1	3	T	T	Distinguish types of inference	
C1C	1	3	T	T	Distinguish deterministic, probabilistic causation	
C1D	1	3	T	T	Determine if event is repeatable or condition is switchable	
C1E	1	3	T	T	Determine if study is repeatable	
C1F	1	3	T	T	Distinguish Association-Causation in time-independent studies	
C1G	1	3	T	T	Distinguish Association-Causation in time-based studies	
C1H	1	3	T	T	Distinguish Association-causation: Likely/risk/can expect	
C1I	1	2	T	T	Determine effect of confounder on a statistic	
C1J	1	2	R	D	Identify which definition gives a higher count or total	
C1K	1	1	A	D	Calculate effect of grouping on counts	
C1L	1	3	A	T	Distinguish causal phrases	
C1M	1	3	A	T	Distinguish different sense of "Can"	
C1N	1	3	T	T	Distinguish common cause, confounder and mechanism	
C1O	1	1	R	R	Randomness: Law of Very Large Numbers	
C1P	1	3	E	D	Distinguish major types of error or bias	
C1Q	1	3	T	T	Distinguish Confounding, Assembly, Randomness and Error	

ID	MC	Mth	care	stat	**CHAPTER 2:**	**15**
C2A	1	1	A	C	Calculate size of comparison for different types	
C2B	1	2	C	C	Distinguish appropriate comparison grammar	
C2C	1	2	C	C	Compare percentages and numbers with units	
C2D	1	1	A	C	Compare 2#: choose base & type compare	
C2E	1	1	A	D	Calculate effect of definitions on averages	
C2F	1	3	T	T	Distinguish Longitudinal vs. cross-sectional	
C2G	1	3	T	T	Distinguish Experiment vs. observational	
C2H	1	3	T	T	Distinguish Controlled vs. uncontrolled	
C2I	1	3	T	T	Hypothetical thinking: Plausible confounders	
C2J	1	2	T	T	Estimate the implications of statistics if true	
C2K	1	3	R	D	Estimate effect of randomness given sample size	
C2L	1	1	E	D	Calculate the effect of non-response bias	
C2M	1	3	E	D	Distinguish types of bias	
C2N	1	2	E	D	Estimate result of question change	
C21	0	2	A	C	Write out different types of comparisons	

The 23 exercises in Chapter 3 deal with measurements.

ID	MC	Mth	care	stat	**CHAPTER 3: MEASUREMENTS**	23
C3A	1	1	A	D	Calculate & compare ranks from scores	
C3B	1	1	A	D	Calculate percentiles in different size groups	
C3C	1	1	A	D	Identify which percentile, score or rank is higher	
C3D	1	1	A	D	Identify which mean is higher in closely related groups	
C3E	1	1	A	D	Compare averages from extremes of a distribution	
C3F	1	1	C	D	Calculate weighted average given subgroup averages	
C3G	1	1	A	D	Calculate mean, median & mode given data values	
C3H	1	1	C	S	Calculate/compare weighted average before/after standardization	
C3I	1	1	C	S	Standardize measures for binary confounder	
C3J	1	1	C	S	Calculate & compare Z-scores	
C3K	1	1	C	S	Calculate & compare Normalized scores	
C3L	1	1	R	D	Calculate Prediction Intervals	
C3M	1	1	R	S	Calculate & compare Coefficients of Variation	
C3N	1	1	R	S	Calculate & compare Effect Sizes	
C3O	1	1	A	D	Predict outcome given regression & predictor	
C3P	1	1	A	D	Calculate correlation from slope and std deviation	
C3Q	1	1	A	D	Compare correlations.	
C3R	1	1	A	S	Calculate "percentage explained" from correlations	
C3S	1	1	A	D	Predict outcome given correlation, SD & predictor	
C3T	1	1	R	D	Calculate Prediction Interval from correlation, SD & predictor	
C3U	1	2	E	D	Excel: Identify effect of outlier on slope and correlation	
C31	0	2	A	C	Write out comparisons of numbers	
C32	0	2	A	C	Write out comparisons of statistics that have units	

The 22 exercises in chapter 4 deal with ratios: calculating, describing and communicating percentages and rates. Ideally, some – if not most – of this material would be covered at the school level.

ID	MC	Mth	care	stat	**CHAPTER 4: DESCRIBING RATIOS**	22
C4A	1	2	C	P	Identify part in questions using "What percentage..."	
C4B	1	1	C	P	Calculate percentages from count tables: % grammar	
C4C	0	1	C	P	Create 100% row, column and total tables from counts	
C4D	0	1	C	P	Create half tables of percentages from count data.	
C4F	1	2	C	P	Identify part in statements using percent grammar	
C4G	1	2	C	P	Identify part in statements using percentage grammar	
C4H	1	2	C	P	Convert statements: percentage to percent grammar	
C4I	1	2	C	P	Convert statement: percent to percentage grammar	
C4J	1	2	C	P	Identify part in questions using "What is the percentage..."	
C4K	1	1	C	P	Calculate percentages from counts: percentage grammar	
C4L	1	2	C	P	Identify part in statements: percent or percentage grammar	
C4M	1	2	C	P	Identify part in questions: percent or percentage grammar	
C4N	1	1	C	P	Calculate percentages from count tables: % or percentage questions	
C4O	1	2	C	P	Identify part in phrase-based rate statement	
C4P	1	2	C	P	Identify equivalent phrase-based rate statement given "Per" ratio	
C4Q	1	2	C	P	Identify part in clause-based rate statement	
C4R	1	2	C	P	Translate between phrase-based & clause-based rate grammar	
C4S	1	2	C	P	Identify part(s) and whole(s) in chance grammar statements	
C4T	1	2	C	P	Translate between chance and percent/percentage grammar	
C4U	1	2	C	P	Translate between rate and chance grammar	
C41	0	2	C	P	Decode part/whole in descriptions and questions	
C42	0	2	C	P	Write descriptions of ratios presented in statements, charts or tables.	

The 20 exercises in chapter 5 deal with comparisons of rates (rates and percentages).

ID	MC	Mth	care	stat	CHAPTER 5: COMPARING RATIOS	20
C5A	1	1	C	S	Calculate percentage attributable given count data	
C5B	1	1	C	S	Calculate percentage attributable from percentage/rate data.	
C5C	1	1	C	S	Calculate cases attributable given rates and # of exposure cases	
C5D	1	2	C	L	Decode part and base in comparison using percentage grammar	
C5E	1	2	C	L	Determine if common or distinct part: percentage grammar	
C5F	1	2	C	L	Decode part and base in comparison using rate phrase grammar	
C5G	1	2	C	L	Determine if common or distinct part: rate phrase grammar	
C5H	1	2	C	L	Decode part and base in comparison using rate clause grammar	
C5I	1	2	C	L	Determine if common or distinct part: rate clause grammar	
C5J	1	2	C	L	Decode part and base in comparison using chance grammar	
C5K	1	2	C	L	Determine if common or distinct part: chance grammar	
C5L	1	2	C	L	Decode part and base in comparison using likely grammar	
C5M	1	2	C	L	Determine if common or distinct part: likely grammar	
C5N	1	2	C	L	Determine if common or distinct part: table or graph	
C5O						
C5P	1	2	C	S	Identify direction given involvement, Bayes compare or data	
C5Q	1	2	C	S	Generate Bayes comparison given rates, charts or row/col tables	
C5R	1	1	C	S	Calculate 4th rate given other three	
C5S	1	1	C	S	Calculate relative risk given counts or ratios	
C51	0	2	C	L	Decode part/whole/test/base in written comparisons	
C52	0	2	C	L	Write comparisons of ratios data in statements, graphs & tables	

The 14 exercises in Chapter 6 involve interpreting and standardizing rates and percentages.

ID	MC	Mth	care	stat	CHAPTER 6: INTERPRETING RATIOS	14
C6A	1	2	A	L	Compare inverse percentages on higher support	
C6B	1	2	A	L	Compare two related three-factor percentages	
C6C	1	2	A	L	Compare percentages involving related wholes	
C6D	1	1	A	L	Compare shares from extremes of a distribution	
C6E	1	1	A	L	Calculate prediction and explanation in 2x2 tables	
C6F	1	1	A	L	Calculate prediction given prevalence & accuracy	
C6G	1	1	C	L	Identify which percentage is higher in closely related groups	
C6H	1	1	C	L	Compare ave % from extremes of a distribution	
C6I	1	1	C	P	Calculate weighted ave % given subgroup averages	
C6J	0	1	C	S	Standardize percentages for effect of binary confounder	
C6K	0	1	C	S	Standardize rates for effect of binary confounder	
C6L	0	1	C	S	Standardize Percentage & Cases Attributable for confounder effect	
C6M	1	1	C	S	Identify change conditions in standardized associations	
C6N	1	1	C	S	Predicting type of change after standardization	

The 19 exercises in Chapter 7 involve randomness: StatLit (CARE) = R. Note that the last three involve the influence of confounding (standardization) on statistical significance. (Stat=S).

ID	MC	Mth	care	stat	**CHAPTER 7: RANDOMNESS AND CHANCE**	19
C7A	1	1	R	R	Calculate Expected Value	
C7B	1	3	R	D	Distinguish different types of chance	
C7C	1	1	R	R	Calculate Response to Sensitive Issues	
C7D	1	1	R	R	Calculate Number using Capture-Recapture	
C7E	1	1	R	R	Calculate chance of rare event; use Law of Very Large Numbers	
C7F	1	1	R	R	Calculate Regression to the Mean	
C7G	1	1	R	R	Calculate ME, CI & Sample size for Percentages	
C7H	1	1	R	R	Calculate ME, CI and Sample Size for Small Rates	
C7I	1	1	R	R	Calculate ME, CI & Sample Size for Averages	
C7J	1	1	R	R	Generate CI & Stat. Significance for two proportions	
C7K	1	1	R	R	Generate CI & Stat. Significance for two small rates	
C7L	1	1	R	R	Generate CI & Stat. Significance for two averages	
C7M	1	1	R	R	Determine if stat significant difference given ME	
C7N	1	1	R	R	Calculate sample size to make difference stat signfcnt (%)	
C7O	1	1	R	R	Calculate sample size to make difference stat signfcnt (rates)	
C7P	1	1	R	R	Calculate sample size to make difference stat signfcnt (averages)	
C7Q	1	1	R	S	Generate confounder effect on Stat. Significance for percentages	
C7R	1	1	R	S	Generate confounder effect on Stat. Significance for small rates	
C7S	1	1	R	S	Generate confounder effect on Stat. Significance for averages	

APPENDIX B: BASIC STATISTICAL LITERACY EQUATIONS

Eq	DESCRIPTION
1	Simple Difference Comparison: Test - Base
2	Simple Ratio Comparison: Test/Base
3	Relative Difference Comparison: (Test - Base)/Base
4	Percentage Difference Comparison = 100% * Relative Difference
5	Percentile = 100[(RankAscending - 1) / (#Subjects - 1)]
6	RankAscending = (#Subjects - RankDescending) + 1
7	Percentile = 100{1 - [(RankDescending - 1) / (#Subjects - 1)]}
8	Mean = Sum of values / # of subjects
9	Ave1 = [F1b*M1b + (1-F1b)*M1a] = [M1a + F1b(M1b-M1a)]
10	Ave2 = [F2b*M2b + (1-F2b)*M2a] = [M2a + F2b(M2b-M2a)]
11	DifferenceBeforeStandardization = Ave2 - Ave1
12	Standardization: F1b = F2b = Fstd
13	DifferenceAfterStandardization = Ave2std - Ave1std
14	% difference explained by confounder: 100% (Before-After)/Before
15	StdYvalue = ActualYvalue - Slope*(ActualXvalue - AveXvalue)
16	Range = Maximum - Minimum
17	Interquartile Range = 75th percentile score - 25th percentile score
18	Z = (Individual Score - Mean) / (Standard deviation)
19	Standardized Score = Target Mean + Z * Target Standard Dev.
20	Effect Size = (TestMean - BaseMean) / PooledStdDev.
21	Coefficient of Variation = Standard Deviation / Mean
22	Skewness = (Mean - Median) / Standard Deviation
23	Correlation = Standardized Slope = Slope (SDx/SDy)
24	% Attributable to Exp = 100%(ExpRate - CntrlRate)/ExpRate
25	Cases Attributable to Exposure = (% Attributable)(#Exposures)
26	% Attributable to Exposure = 100% (RR-1) / RR
27	Cases attributable to exposure = [(RR-1)/RR](# exposed)
28	Over-Involvement: If $P(C \mid A) > P(C \mid \sim A)$ then $P(A \mid C) > P(A \mid \sim C)$
29	Bayes Rule: $[P(C \mid A) / P(C \mid W)] = [P(A \mid C) / P(A \mid W)]$
30	Conditional Probability: $P(AB \mid C) \le P(A \mid BC)$.
31	Simple Probability: P(A)=1/N if A is one of N equally-likely outcomes.
32	Multiple tries: If P(A) = p, then $P(A \mid n \text{ independent tries}) = p^n$
33	Law of Large #: If p(A) = 1/n, then $P(A \mid n \text{ independent tries}) > 50\%$
34	Estimated Population statistic = PointEst = Random sample statistic
35	Expected # of Sample Successes = P times Sample size
36	Percentage who did X = (Number of Yes - 50%*N)/(50% * N)
37	Guessing Adjusted Score = [# Right - (N / k)] [k / (k-1)]
38	Guessing Adjusted Score = # Right - [#Wrong / (k-1)]
39	Ntotal = N1stCapture / Fraction recaptured
40	95% Confidence interval: Point Est ± 95% margin of error
41	Proportions: 95% Exact margin error = $2\sqrt{[p(1-p)/n]}$
42	Measures: 95% Exact margin of error = $2s/\sqrt{n}$
43	Proportions: 95% Conservative margin of error = $1/\sqrt{n}$
44	Proportions: N = $[1/\text{ErrorAllowable}]^2$ with 95% confidence
45	Measures: N = $[2s/\text{ErrorAllowable}]^2$ with 95% confidence
46	Subgroup ME = Group ME / SQRT(# Subgroup / # Group)
47	Proportions: 95% Conservative Conf. Intrvl: $p \pm 1/\sqrt{n}$
48	Measures: 95% Confidence Interval: mean ± $[2 s/\sqrt{n}]$