TRAINING TEACHERS TO DEVELOP STATISTICAL THINKING

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In this paper I argue that to improve teachers' statistical content and pedagogical content knowledge, teachers need to experience the game of statistics, build key statistical concepts related to transnumeration thinking, reasoning with statistical models, and consideration of variation, and understand how students develop their statistical reasoning. The implication of requiring teachers to have substantive and deep knowledge of statistics is discussed.

INTRODUCTION

Decision-making in society and learning about the world are increasingly being based on evidence from data. Statistical methods and ways of thinking are pervading a diverse range of human endeavours such as psychology, government policy, engineering, health sciences and sustainable environments. All these endeavours are using data to extract meaning and insight about real context and real situations. This use of statistics, however, is not reflected in classrooms. Currently, many students are taught mean, median, mode, and graphs with an emphasis on how to construct them rather than how to use them to think with data (Friel, O'Connor & Mamer, 2006). Students are given little opportunity to play, what I like to call, *the game of statistics* in an exploratory data analysis environment. Moreover, knowledge of how to build students' conceptual understanding of statistical ideas is emergent and, thus, not yet fully understood by researchers and teachers. Hence statistical conceptual pathways such as building students' inferential reasoning or distributional reasoning do not underpin the curriculum or teaching.

In the 1990s there was a strong call from some prominent statisticians to develop students' statistical thinking (e.g., Moore, 1990), resulting in the American Statistical Association setting the agenda for the future of statistics education by promoting three elements of practice: Emphasise statistical thinking; use more data and concepts, less theory and recipes; and foster active learning. By 2000 statistics education researchers took up the challenge to focus research on statistical thinking, reasoning, and literacy (Ben-Zvi & Garfield, 2004). In particular, Wild and Pfannkuch (1999) developed a four-dimensional framework that attempted to characterise statistical thinking in empirical enquiry.

Using part of this framework and statistics education research I will argue that there are some fundamental learning experiences that teachers should have to develop their statistical thinking. One fundamental learning experience is learning the game of statistics (Dimension 1 of the framework: the investigative cycle) and another is the building of sufficient statistical concepts (Dimension 2 of the framework: fundamental statistical thinking elements) for teachers to appreciate some "big ideas" of statistics and to understand how students develop these ideas. There are four main assumptions behind the argument. The first assumption is that teacher educators, that is, the teachers of the teachers, are well versed in current statistics education research, have good statistical content and pedagogical content knowledge, and have an empirical, rather than a mathematical, approach to statistics. The second assumption is that improving teachers' knowledge will improve student learning (Hill, Rowan & Ball, 2005), the third assumption is that technology such as Tinkerplots (Konold & Miller, 2004) and Fathom (Key Curriculum Press Technologies, 2000) are an integral part of school-level statistics learning, and the fourth assumption is that genuine data is always used. Basing my argument on Dimensions 1 and 2 of the framework, I will elaborate on the types of learning experiences that teachers need to facilitate students' statistical learning and thinking in the classroom.

These fundamental learning experiences are not situated in the real setting of the statistician in the field but in the classroom investigations and experiences that school students would undertake as part of their statistical course. Effective teacher learning, according to Moore, Cobb, Garfield and Meeker (1995), and Ball and Cohen (1999), includes teachers participating in activities as students in simulated classroom settings, reflecting on and studying the theoretical basis or rationale for the teaching method from a learner and teacher perspective,

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observing demonstrations by experts, the teacher educators, and having the time to learn in and from practice within a professional learning community. A focus on learning about levels of students' reasoning (see Watson, 2006) should be within the reflections on the theoretical basis for the teaching method as Wilson and Berne (1999), in a review of research on teacher learning, contend that improvement in teacher and student knowledge occurs when the teacher is able to analyse students' thinking guided by an instructional framework. Although Hill, Rowan and Ball (2005) report that improving teacher knowledge will improve student knowledge, there is no empirical research evidence that teachers working on statistical investigations leads to improved teacher statistical thinking that, in turn, will improve their students' thinking. Hence the claims made in this paper remain speculative.

LEARNING THE GAME OF STATISTICS

The roots of the statistical discipline lie in empirical enquiry (Moore, 1990). The purpose of the enquiry is to learn more in the context sphere by collecting data, and unlocking the stories in the data. Underpinning this learning from data is the ability to explore and interrogate data. The exploratory data analysis environment is a key learning experience for understanding the power and limits of data-based arguments.

Experiencing the whole empirical enquiry cycle, from understanding the contextual situation, formulating problems, defining variables, determining methods of measurement, designing methods of data collection, collecting data, and so forth, is a fundamental learning experience (Konold & Higgins, 2002). Both sample survey and experimental processes of enquiry should be understood from the perspectives of investigator and participant. Some key learning experiences in sample surveys include using different sampling methods, wording of questions, and methods of measurement to learn about bias and data quality. For experiments key learning experiences should involve simple and randomised experiments. An example of a simple experiment is testing to see which type of paper ball, copier or tissue, dropped from a certain height is better at getting close to a target. Issues that need to be discussed are: Sources of variation such as height, type of surface onto which paper balls are dropped, wind effect; how to measure *close to a target*; how many times should the balls be dropped to draw a conclusion; and what can be concluded from the experiment. Parts of this simple experiment could be randomised, but this would add another level of complication in a learning situation. An example of a randomised experiment is determining whether bottled water tastes better than tap water. Basic experimental design principles such as controlling sources of variation, using randomisation to balance out sources of variation that cannot be controlled, and blinding can be elicited from teachers before conducting such an experiment.

Intrinsic to the process of enquiry is experiencing the exploration of multivariate data sets. Learning to be a data detective by wondering whether some factors might explain differences between two groups or whether there is a relationship between two variables is part of learning the game of statistics. The emphasis should be on teachers posing their own questions about the data, interrogating the data, and learning new information about the real world from the data. Use of technology such as Tinkerplots (Rubin & Hammerman, 2006) or Fathom (Biehler, 2006) can quickly enculturate teachers into unlocking stories in the data whereas in a non-technological environment multivariate datacards can be used as a substitute (Chick, Pfannkuch & Watson, 2005). Even when exploring given multivariate datasets, the role the empirical enquiry cycle plays in teachers' abilities to interrogate the data should be highlighted through appreciating the need to: Have contextual knowledge about the situation since data have their own literature base; understand the design of the study; and to know where the data originated, and how they were measured and collected.

From a teaching perspective, if students are taught parts of the cycle in isolation they are often unable to synthesise an investigation into a coherent whole. Teaching within the cycle is possible, and when necessary, lessons can be focused on particular stages of the cycle such as the analysis stage. Such a teaching approach provides students with an awareness of the purpose of statistics. The teacher educator has an important role to play by demonstrating and making teachers aware of how to facilitate such a teaching approach.

If teachers experience school-type investigations and share with other teachers in their class the problems they encountered and what they learned from the investigations, then they may appreciate the intricacies involved in the process of investigation and how their students might interact with similar investigations. Being aware of how they and other teachers reason and think when conducting investigations may improve their pedagogical content knowledge. However, as Makar and Confrey (2004) discovered, influencing teachers' statistical reasoning is complex, but they believed that the involvement of teachers in investigations could expand their view of statistics and data as well as understanding of concepts such as distribution and variability.

BUILDING STATISTICAL CONCEPTS

Teacher educators need not only to build teachers' statistical concepts but also to make teachers aware of how students' conceptual understanding may develop. Many researchers (e.g., Rubin & Hammerman, 2006) have found that teachers' thinking is not much different from students, and hence the onus is on teacher educators to link, for the teachers, how they are thinking to the ways in which students think. Three fundamental thinking elements from Dimension 2 of the framework should be encouraged in teacher education: transnumeration, reasoning with statistical models, and consideration of variation. I will now discuss the types of experiences teachers should have to develop these elements of statistical thinking.

Transnumeration. Wild and Pfannkuch (1999) coined the word transnumeration to refer to an element of thinking whereby ways of representing data are changed to engender more understanding of the stories within the data. Transnumeration occurs throughout the enquiry cycle from encapsulating real notions such as *prompt service* into data that can be collected and looking at multiple representations of the data to communicating the findings in language and displays that have meaning for the intended audience.

An example of transnumeration is changing raw data into plots and thereby learning that different types of representation reveal different aspects of the story within the data and that from these representations a story can be synthesised (Shaughnessy & Pfannkuch, 2002). Another example of transnumeration is learning that raw data can be recategorised to learn and gain more knowledge from the data. For example, if the raw data list favourite activities as TV, board games, rugby, netball, reading, swimming (Chick, Pfannkuch & Watson, 2005) then these activities could be recategorised as active and sedentary, or team and individual pursuits, or competitive and non-competitive, and so forth. The recategorisation of the data enables more information and more insight into the data. From the student perspective teachers need to learn how students will intuitively represent data and how they as teachers can scaffold students to consider other representations, which will allow them to learn more from the data. For example, Konold and Higgins (2003) demonstrate how students can be scaffolded from their intuitive individual case bar plots to dot plots, while Bakker and Gravemeijer (2004) explain how to enable students to make the transition from dot plot to box plot. Teachers need to have this basic pedagogical content knowledge.

Reasoning with statistical models. In its most basic form reasoning with statistical models includes reasoning from statistical plots such as bar plots, dot plots, histograms, box plots, scatter plots, two-way tables of counts. Each type of plot structurally organises the data and is dependent on the type of data, qualitative and quantitative. In particular, teachers need to experience and be aware of how students build the following two intertwined concepts: aggregate-based reasoning and how to reason with different types of plots.

According to Konold and Higgins (2003) aggregate-based reasoning or reasoning from the whole distribution is a concept that is built up though a four-fold schema of viewing data initially as a pointer (e.g., reminder that students measured heights), then as an individual case (tallest girl was Poppy), then as a classifier (five students had a height between 150 and 160cm), and finally as an aggregate (students typically had a height between 150 and 170cm). The aggregate view focuses on overall characteristics of a distribution such as variability around a centre. But reasoning from distributions involves a dual process whereby the reasoner moves seamlessly between the aggregate properties and individual cases. Hence students must have their attention shifted from their intuitive individual case focus to the notion of distribution such as growing a sample (Bakker, 2004) and then be enculturated into the dual perspectives. The building of the concept of the mean or average should be integrated into learning to reason from distributions with teachers cognisant of the research of Mokros and Russell (1995), Watson (2006), and Konold and Pollatsek (2002).

Reasoning from data plots requires the ability to notice, decode, assess, and judge and to express verbally and in writing the messages inherent in the plot. Biehler (1997) noted that teachers and researchers found such reasoning difficult. Developing teachers' statistical language, argumentation, and communication is paramount in learning how to reason from plots. For example, Pfannkuch (2006) listed ten elements of reasoning associated with reasoning from the comparison of box plots. Teachers need to gain fluency in reasoning from different types of plots and to appreciate how students' language develops. For example, when describing distributions, students' language may develop from bumps to symmetrical and skewed sample distributions, from squashed up to clusters, and from far apart to spread. The enculturation of students into the language of statistics is crucial in conceptual development as the concepts develop with the language. However, such reasoning development requires time and experience in the classroom. Therefore the teacher educator could perhaps focus developing teachers' language on the comparison of dot plots as these plots are particularly easy to interpret and are ubiquitous within the statistics discipline.

Consideration of variation. Recognition that variation plays a major role in determining, handling, and interpreting data is fundamental to teacher experience. In particular, sampling reasoning, sampling variability, inferential reasoning, and consideration of sources of variation are some of the knowledge building blocks.

Watson (2006) developed a six-level hierarchy to explain students' conceptual development of sampling reasoning. Many facets of sampling reasoning such as the notion of a sample, sampling methods, sample size, notion of randomness are within the hierarchy. If teachers know this hierarchy then they will be aware that students' initial ideas of sample are often food samples in supermarkets, which means that their teaching approaches must consider how to build students' ideas towards viewing a sample as a representative part of a whole. Watson (2006, p. 48) suggests that students need to experience sampling in many different contexts including chance settings, opinion surveys, and measurement surveys, since such experiences will build "many associations of different sampling methods for different contexts". Groth and Bergner (2005) demonstrated that teachers had impoverished metaphors for the concept of a sample and suggested that teacher educators needed to design learning pathways that enhanced teachers' knowledge of the concept of statistical sample. They also suggested teachers needed to experience problems that required samples for the purpose of making an inference about the population.

Integrated with the idea of taking a representative sample for inference is the idea of sampling variability. Rubin, Bruce, and Tenney (1991, p. 318) explain that senior high school students in their study tended to believe that a representative sample was one that was sampled correctly and that randomness was "not sufficient to explain sampling variability – some mechanism or bias must be postulated to explain it." Similarly Saldanha and Thompson (2002, p. 264), who focused on building senior students' conceptions about sampling distributions in their study, comment that students "image of sampling did not entail a sense of variability that extended to ideas of distribution." Hence, before students and teachers are introduced to sampling distributions I believe that they must build up images of resampling from populations and that their attention should be drawn to noticing the variability between sample distributions in terms of shape, spread, and sample statistics. Linked to the notion of a sample distribution are the notions of a population and population distribution, both of which should be discussed and understood. The concepts of empirical and theoretical distributions and their relationships to variation, sampling, and inference are also important considerations when designing learning experiences for teachers (Wild, 2006). Another idea to consider when building sampling concepts for statistical inference is the sample size effect. Teachers need to experience sampling variability of sample distributions with the same and different sample sizes with both qualitative and quantitative data. Intrinsic to all these interrelated ideas underpinning statistical inference is the key idea that an inference is made about a population from a representative sample.

As previously noted, teachers need to be familiar with considering sources of variation when designing experiments or surveys. When interpreting data plots, sources of variation can also be considered, when, for example, the pattern in the variability suggests a possible contextual explanation. In a recent *CensusAtSchool* survey in New Zealand of over 30,000 students from 8 to 15 year-olds, it was noticed that the distribution of their heights had spikes occurring at regular intervals. A possible contextual explanation for these spikes was that the students had rounded their heights to the nearest 5cm. In particular, a large spike at 1m perhaps was a desire on the part of some young students to achieve that height. The variation in heights is an observable reality and can be explained by factors such as age, gender, and nationality. Another source of variation may be induced by the actual taking of a sample, and hence notions of sampling variability, confidence intervals, and probability need to be developed.

CONCLUSION

I have argued that teachers as part of their content knowledge must learn to play the game of statistics in empirical enquiry and build key statistical concepts related to transnumeration thinking, reasoning with statistical models, and consideration of variation. For pedagogical content knowledge teachers should be cognisant of the research on how students develop their statistical reasoning, and know how to use that knowledge to scaffold student learning. In order to enculturate students into a statistical community of practice and thinking, teachers' statistical language must be fully developed to communicate and model ways of unlocking stories from data and statistical argumentation. The challenge for teacher educators is to find ways of improving teachers' statistical content and pedagogical content knowledge. The challenge for researchers is to communicate their findings in ways that will impact on improving teachers' practice and students' learning. The challenge for all the statistical community of statisticians, researchers, teacher educators, teachers, curriculum developers, textbook writers, and technology developers is to find ways of continually growing teachers' statistical learning.

To be a teacher of statistics is to realise that one is not teaching a branch of mathematics but that one is teaching a discipline that has its own independent intellectual method. Students are now living in a society that demands evidence-based arguments and decisions. Therefore teachers play a crucial role in developing students' statistical thought processes. The implications of requiring teachers to have substantive knowledge of statistics are founded on societal and political goals of education. First, statistical thinking or reasoning or literacy needs to be recognised as a key educational goal for all students. Second, statistics needs to be valued as a distinct discipline. Finally, resources need to be put into more statistics education research to understand how to develop students' statistical concepts and thinking.

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