

Statistics and Modelling: Probability Unit Plan, Level 8

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Note: You may want to supplement the exercises with further examples from other sources.

1. Probabilities from tables of counts

- Calculate probabilities from a table of counts
- Notation for simple events (i.e. $P(A)$), combined events (i.e. $P(A \text{ and } B)$, $P(A \text{ or } B)$), complementary events (i.e. $P(\text{not } A)$)
- Develop formula for $P(A \text{ or } B)$
- Meaning of mutually exclusive (disjoint) events

Students could attempt Ex 1, Q1 before any teacher input

Notation could then be introduced before students attempt Ex 1, Q2 – 5

The main new ideas from these questions are the formula for $P(A \text{ or } B)$ and the meaning of mutually exclusive (disjoint) events

2. Conditional probabilities

- Calculate conditional probabilities from tables of counts
- Notation for conditional probability
- Develop formula for $P(A | B)$, using a table of counts
- Calculate conditional probabilities from tables of counts

Students could attempt the Discussion Exercise (p3) before any teacher input

Notation could then be introduced before students attempt Ex 2

In Q1 all parts are conditional probabilities.

Q2 contains a variety of probability questions on simple, combined ('and', 'or'), conditional and complementary events

Q3 reinforces the meaning of mutually exclusive events

Q4 develops the formula for $P(A | B)$

3. Risk

- Establish that risk has the same meaning as probability or proportion
- Calculate the relative risk of an outcome for two groups
- Calculate the percentage change in risk of an outcome for two groups
- Develop strategies for choosing the baseline group for comparative risk calculations

Ex 3 contains questions, mainly about risk, based on two studies. Note that this material is prepared for Level 8 and that risk is introduced at Level 7 (in the draft curriculum).

These two studies provide an extra opportunity to discuss experimental design principles.

4. Independent Events

- Define independence by $P(A | B) = P(A)$
- Check whether two events are independent or not
- Develop result $P(A \text{ and } B) = P(A) \times P(B)$ for independent events

Ex 4 does the independence checking

The Discussion Exercise (p13) develops the $P(A \text{ and } B) = P(A) \times P(B)$ result

Ex 5 provides an opportunity to reinforce and relate ideas covered so far

5. Tables of Counts and Probability Trees

- Construct and use two-way tables of counts and probability tree diagrams for independent and conditional events

When constructing a table from a story, which gives proportions or percentages in categories (rather than counts or frequencies), it is recommended that a table of counts is constructed rather than a table of proportions. This is a change from the previous version of this unit of work. Research has shown that information in a table of counts is more readily understood than information in a table of proportions. It is suggested that a large and easy-to-use number (such as 1000, 10 000, 100 000 or 1 000 000) is used as the sample size. This number does not need to relate to the size of any underlying population.

When calculating a probability of an outcome using a probability tree there is a danger that students will multiply the probabilities on the branches without giving due consideration to the reasons why. The notation of events in the second stage should be labelled as conditional events when the second stage events are not independent of the first stage event.

When second stage events are not independent of the first stage event the multiplication of probabilities is justified by $P(A \text{ and } B) = P(A | B) \times P(B)$.

When second stage events are independent of the first stage event the multiplication is justified by $P(A \text{ and } B) = P(A) \times P(B)$ for independent events.