

# Probability

## 1. Probabilities from Tables of Counts

The table below contains information for sales of 4 bedroom houses in Auckland City in June, July and August, 2004.

### Exercise 1

Selling price	Days on the market			Total
	Less than 30 days	30 – 90 days	More than 90 days	
Under \$300,000	39	31	15	85
\$300,000 - \$600,000	35	45	4	84
Over \$600,000	8	4	0	12
<b>Total</b>	82	80	19	181

Let C be the event that a house sold for under \$300,000,  
D be the event that a house sold for between \$300,000 and \$600,000 (inclusive),  
E be the event that a house sold for over \$600,000,  
F be the event that the sale was made in less than 30 days,  
G be the event that the sale was made in 30 to 90 days (inclusive), and  
H be the event that the sale was made in more than 90 days.

1. For a sale selected at random from these 181 sales, find the probability that the sale was:

(a) under \$300,000 \_\_\_\_\_

(b) made in less than 30 days \_\_\_\_\_

(c) under \$300,000 and made in less than 30 days \_\_\_\_\_

(d) under \$300,000 or made in less than 30 days \_\_\_\_\_

(e) \$300,000 or more \_\_\_\_\_

2. For a sale selected at random from these 181 sales, find the probability that the sale was:

(a) over \$600,000 \_\_\_\_\_

(b) made in 30 to 90 days (inclusive) \_\_\_\_\_

(c) over \$600,000 and made in 30 to 90 days (inclusive) \_\_\_\_\_

(d) over \$600,000 or made in 30 to 90 days (inclusive) \_\_\_\_\_

(e) over \$600,000 and made in more than 90 days \_\_\_\_\_

(f) over \$600,000 or made in more than 90 days \_\_\_\_\_

(g) made in at least 30 days \_\_\_\_\_

3. The following is a student's answer to Question 1(d).

$$\begin{aligned} P(C \text{ or } F) &= \frac{85 + 82}{181} \\ &= \frac{167}{181} \\ &= 0.9227 \text{ (to 4 d.p.)} \end{aligned}$$

Describe why this answer is not correct.

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4. (a) Consider the event C or G (That is, house sold for under \$300,000 or the sale was made in 30 to 90 days (inclusive)).

$$\begin{aligned} \text{Fill in the gaps: } P(C \text{ or } G) &= \frac{85 + 80 - 31}{181} \\ &= \frac{85}{181} + \frac{80}{181} - \frac{31}{181} \\ &= P(\quad) + P(\quad) - P(\quad) \end{aligned}$$

(b) Consider the event D or H (That is, house sold for between \$300,000 and \$600,000 inclusive or the sale was made in more than 90 days).

$$\begin{aligned} \text{Fill in the gaps: } P(D \text{ or } H) &= \frac{84 + 19 - 4}{181} \\ &= \frac{84}{181} + \frac{19}{181} - \frac{4}{181} \\ &= P(\quad) + P(\quad) - P(\quad) \end{aligned}$$

(c) Fill in the gaps: For any two events A and B

$$P(A \text{ or } B) = \underline{\hspace{2cm}} + \underline{\hspace{2cm}} - \underline{\hspace{2cm}}$$

5. Look at your answer to Question 2(e).

(a) Comment on the events E and H.

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(b) E and H are called **mutually exclusive** events or **disjoint** events.

Fill the gap: For any two mutually exclusive events A and B

$$P(A \text{ and } B) = \underline{\hspace{2cm}}$$

### Discussion Exercise

Selling price	Days on the market			Total
	Less than 30 days	30 – 90 days	More than 90 days	
Under \$300,000	39	31	15	85
\$300,000 - \$600,000	35	45	4	84
Over \$600,000	8	4	0	12
<b>Total</b>	82	80	19	181

For the 181 house sales in the above table, what proportion of the houses that sold for over \$600,000 were on the market for less than 30 days?

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## 2. Conditional Probability

### Exercise 2

Selling price	Days on the market			Total
	Less than 30 days	30 – 90 days	More than 90 days	
Under \$300,000	39	31	15	85
\$300,000 - \$600,000	35	45	4	84
Over \$600,000	8	4	0	12
<b>Total</b>	82	80	19	181

1. For the 181 house sales in the above table:

(a) What proportion of the houses that sold for under \$300,000 were on the market for less than 30 days?

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(b) Given that a house sold for between \$300,000 and \$600,000 (inclusive), what proportion were on the market for more than 90 days?

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(c) If a house was on the market for less than 30 days, what proportion sold for over \$600,000?

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(d) Of those on the market for more than 90 days, what proportion sold for under \$300,000?

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2. For the 181 house sales in the above table:

(a) What proportion of the houses took more than 90 days to sell?

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(b) What proportion sold for under \$300,000 given that they were on the market for 30 to 90 days (inclusive)?

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(c) What proportion took less than 30 days to sell or sold for between \$300,000 and \$600,000 (inclusive)?

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(d) Of those that sold for over \$600,000, what proportion were on the market for more than 90 days?

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(e) What proportion sold for between \$300,000 and \$600,000 (inclusive) and took from 30 to 90 days (inclusive) to sell?

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(f) What proportion of the houses took 90 days or less to sell?

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(g) What proportion of the houses that were on the market for more than 90 days sold for over \$600,000?

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(h) What proportion sold for over \$600,000 and took more than 90 days to sell?

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3. Comment on your answers to Questions 2(d) and 2(g).

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4. (a) Fill in the gaps:

Probability a house sold for under \$300,000 given that it sold in less than 30 days

$$= P(\quad | \quad)$$

$$= \underline{\hspace{2cm}}$$

$$= \frac{\cancel{181}}{\cancel{181}}$$

$$= \frac{P(\quad)}{P(\quad)}$$

(b) Fill in the gaps:

Probability a house sold for between \$300,000 and \$600,000 (inclusive) given that it was on the market for between 30 and 90 days (inclusive)

$$= P(\quad | \quad)$$

$$= \underline{\hspace{2cm}}$$

$$= \frac{\cancel{181}}{\cancel{181}}$$

$$= \frac{P(\quad)}{P(\quad)}$$

(c) Fill in the gaps: For any two events A and B,

$$P(A | B) = \frac{P(\quad)}{P(\quad)}$$

### 3. Risk

#### Exercise 3

##### Study 1

In 1988 the results of the Physicians' Health Study Research Group study were reported in the *New England Journal of Medicine*. In this study 22 071 male physicians (aged from 40 to 84) were randomly assigned to two groups. One group took an aspirin every second day and the other group took a placebo, a pill with no active ingredient which looked just like an aspirin. The participants did not know whether they were taking aspirin or the placebo.

After five years the number of participants in each group who had had a heart attack was recorded. The results are shown in the table below.

<b>Treatment</b>	Heart attack	No heart attack	<b>Total</b>
Aspirin	104	10 933	11 037
Placebo	189	10 845	11 034
<b>Total</b>	293	21 778	22 071

1. All parts of this question apply to Study 1.

(a) For those in the aspirin group:

(i) The proportion who had a heart attack = \_\_\_\_\_  
= \_\_\_\_\_

(ii) The probability that a randomly selected participant had a heart attack  
= \_\_\_\_\_

(iii) The percentage who had a heart attack = \_\_\_\_\_

(iv) The risk of having a heart attack = \_\_\_\_\_

(v) Write this risk as a rate per 1000 participants \_\_\_\_\_

(vi) Write this risk as a rate per 10 000 participants \_\_\_\_\_

(b) (i) For those in the placebo group, the risk of having a heart attack

= \_\_\_\_\_

= \_\_\_\_\_

(ii) Write this risk as a rate per 1000 participants \_\_\_\_\_

(iii) Write this risk as a rate per 10 000 participants \_\_\_\_\_

(c) (i) Calculate the relative risk of having a heart attack using the risk for the placebo group as the denominator (i.e., as the baseline risk).

Relative risk = \_\_\_\_\_

= \_\_\_\_\_

(ii) Interpret this relative risk.

For male physicians aged 40 to 84, \_\_\_\_\_

\_\_\_\_\_  
\_\_\_\_\_

(iii) Calculate the relative risk of having a heart attack using the risk for the aspirin group as the baseline risk.

Relative risk = \_\_\_\_\_

= \_\_\_\_\_

(iv) Interpret this relative risk.

For male physicians aged 40 to 84, \_\_\_\_\_

\_\_\_\_\_  
\_\_\_\_\_

(v) Which group is more appropriate as the baseline group? Briefly explain.

\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_



- (d) (i) Do male physicians aged 40 to 84 who take aspirin every second day have an **increased** or **decreased** risk of having a heart attack compared to those who take a placebo?

Circle one: **increased** **decreased**

- (ii) Calculate the percentage change in risk relative to the baseline (placebo group) risk.

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- (iii) Interpret this percentage change in risk.

For male physicians aged 40 to 84 \_\_\_\_\_

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## Study 2

In 2006 the results of a study carried out among 132 271 Jewish children born in Israel during 6 consecutive years in the 1980s were published in the *Archives of General Psychiatry*. The objective of the study was to examine the relationship between father's age at birth of child (offspring) and their risk of autism.

The offspring were assessed for autism at age 17 years. The results are shown in the table below.

Father's Age Group	Autism	No autism	Total
15 – 29	34	60 654	60 688
30 – 39	62	67 211	67 273
≥ 40	14	4 296	4 310
<b>Total</b>	110	132 161	132 271

2. All parts of this question apply to Study 2.

(a) For offspring from fathers aged 15 to 29 at the birth of their child:

(i) The proportion who had autism = \_\_\_\_\_

= \_\_\_\_\_

(ii) The probability that a randomly selected offspring had autism

= \_\_\_\_\_

(iii) The percentage who had autism = \_\_\_\_\_

(iv) The risk of having autism = \_\_\_\_\_

(v) Write this risk as a rate per 10 000 offspring \_\_\_\_\_

(b) (i) For offspring from fathers aged 30 to 39 at the birth of their child, the risk of having autism

= \_\_\_\_\_

= \_\_\_\_\_

(ii) Write this risk as a rate per 10 000 offspring \_\_\_\_\_

(c) (i) For offspring from fathers aged 40 or more at the birth of their child, the risk of having autism

= \_\_\_\_\_

= \_\_\_\_\_

(ii) Write this risk as a rate per 10 000 offspring \_\_\_\_\_

(d) Using the risk for fathers in the 15 – 29 year age group as the baseline:

(i) Calculate the relative risk for fathers in the 30 – 39 year age group of having autistic offspring.

Relative risk = \_\_\_\_\_

= \_\_\_\_\_

(ii) Interpret this relative risk.

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(iii) Calculate the relative risk for fathers in the 40 or more year age group of having autistic offspring.

Relative risk = \_\_\_\_\_

= \_\_\_\_\_

(iv) Interpret this relative risk.

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In parts (e) and (f), use the 15 – 29 year age group as the baseline group.

- (e) (i) Do fathers aged 30 to 39 have an **increased** or **decreased** risk of having autistic offspring compared to those aged 15 to 29?

Circle one: **increased** **decreased**

- (ii) Calculate the percentage change in risk relative to the baseline risk.

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- (iii) Interpret this percentage change in risk.

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- (f) (i) Do fathers aged 40 or more have an **increased** or **decreased** risk of having autistic offspring compared to those aged 15 to 29?

Circle one: **increased** **decreased**

- (ii) Calculate the percentage change in risk relative to the baseline risk.

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- (iii) Interpret this percentage change in risk.

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## 4. Independent Events

If A and B are independent events then  $P(A | B) =$  \_\_\_\_\_

### Exercise 4

Selling price	Days on the market			Total
	Less than 30 days	30 – 90 days	More than 90 days	
Under \$300,000	39	31	15	85
\$300,000 - \$600,000	35	45	4	84
Over \$600,000	8	4	0	12
<b>Total</b>	82	80	19	181

1. Recall: C is the event that a house sold for under \$300,000 and F is the event that the sale was made in less than 30 days. Are events C and F independent?

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2. Recall: E is the event that a house sold for over \$600,000 and H is the event that the sale was made in more than 90 days. Are events E and H independent?

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### Discussion Exercise

Fill the gaps: For events A and B,  $P(A|B) = \frac{P(\quad)}{P(\quad)}$

If A and B are independent events,  $P(A|B) = P(\quad)$

So if A and B are independent events,  $\frac{P(\quad)}{P(\quad)} = P(\quad)$

That is, if A and B are independent events  $P(\quad) = P(\quad) \times P(\quad)$

### Exercise 5

1. For events A and B,  $P(A) = 0.8$ ,  $P(B) = 0.5$  and  $P(A \text{ or } B) = 0.9$ .

(a) Calculate  $P(A \cap B)$

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(b) Calculate  $P(A | B)$

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(c) Are A and B independent events? Justify your answer.

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2. For events A and B,  $P(A) = 0.3$ ,  $P(B) = 0.4$  and  $P(A | B) = 0$ .

(a) Are A and B independent events? Justify your answer.

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(b) Are A and B mutually exclusive events? Justify your answer.

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3. Events A and B are independent. Also  $P(A | B) = 0.4$  and  $P(B) = 0.6$ .

(a) State  $P(A)$

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(b) Calculate  $P(A \text{ and } B)$

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4. Events A and B are mutually exclusive. Also  $P(A) = 0.4$  and  $P(B) = 0.3$ .

(a) State or calculate  $P(A | B)$

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(b) Calculate  $P(A \text{ or } B)$

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## 5. Tables of Counts and Probability Trees

### Blood Group Systems Example: Example 4.7.3 from Chance Encounters (p 179)

There are a large number of genetically based blood group systems that have been used for typing blood. Two of these are the Rh system (with blood types Rh+ and Rh-) and the Kell system (with blood types K+ and K-). It is found that any person's blood type in any one system is independent of his or her blood type in any other.

It is known that, for Europeans in New Zealand, about 81% are Rh+ and about 8% are K+. If a European New Zealander is chosen at random, what is the probability that he or she is either positive in both systems or negative in both systems?

#### Table of Counts Approach

Rh system	Kell system		Total
	K+	K-	
Rh+			
Rh-			
<b>Total</b>			

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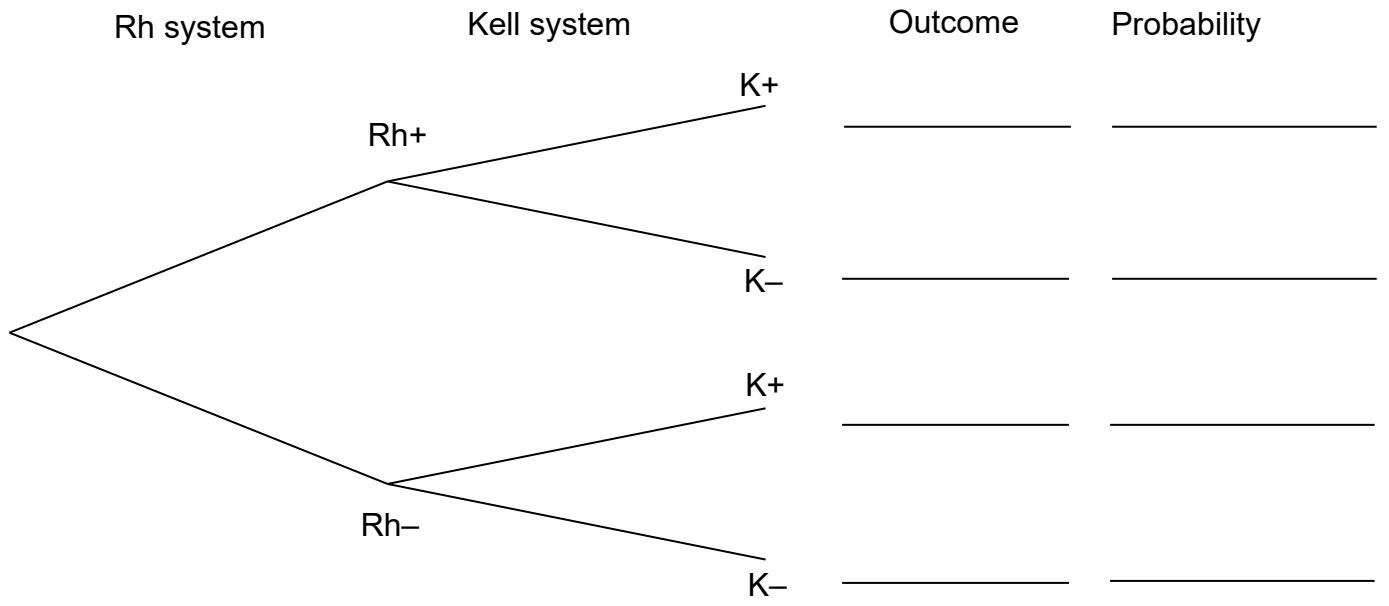
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# Probability Tree Approach



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**Imperfect Testing Procedures Example:**

The ELISA test is used as a screening test for HIV. The test, however, is **not** a perfect one.

For people with HIV: 99.7% test positive

For people without HIV: 0.3% test positive

It is estimated that 0.1% of the New Zealand population have HIV.

**Question:** Suppose that a person is picked at random from New Zealand.

What is the probability of having HIV given that the test is positive?

**Table of Counts Approach**

Disease status	Test result		Total
	Positive	Negative	
HIV			
Not HIV			
<b>Total</b>			

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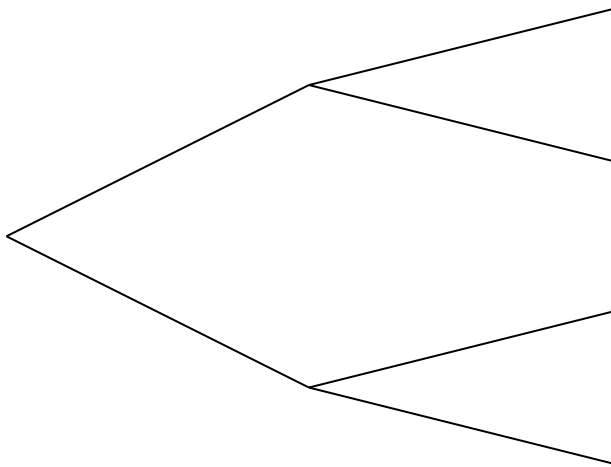
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## Probability Tree Approach

Outcome

Probability



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