Sample-to-population Inference progressions – TEACHING

Curriculum Level 5 (Year 10)

See Census@School lesson progression (part 1, part 2)

New ideas

- Comparative investigative question "*I wonder if A tends to be bigger than B in the population?*" or equivalent with variable, groups, population and direction clear.
  "tend to" is used as we are interested in comparing the overall distributions of each group, and looking for evidence if most of one group is larger than most of the other group (eg: most boys are taller than most girls – not all boys being taller than all girls).
- Using a sample to make statements about the population (see C@S lessons) (draw conclusion = answer investigative question)
  It is important the students experience a "population" in the form of data cards, and have an opportunity to experience answering an investigative question using the whole population (their first response given a bag of cards is generally tipping the whole bag out) – the "AHA!" moment when one of the groups ask "Do we really have to sort them all Miss?" is the teaching moment you are looking for to introduce sampling.
- Different samples give different results → sampling variation
  Again, it is important that this is reinforced to students by physically taking samples from populations and comparing the results of the different samples within the class.
- When comparing two samples from A and B, we can say that A tends to be bigger (smaller) than B back in the population if the differences in the samples is big enough to show up consistently if we repeatedly sampled
  → introduce **Level 5 guide** to "making the call" (based on relative position of medians and spread (middle 50%)).
  Students develop this guide through class investigation, exploring two situations – one where they can make the call and one where they cannot. Students compare multiple samples from these two situations and notice consistencies and differences. Full lesson progression is available on Census@School *(part 1, part 2)*
- We should be always working with a representative sample obtained using a random sampling method from a clearly defined population to make our inferences.
  Be aware that with data sets such as All Blacks vs Wallabies we often find it difficult to describe the sample and the population. Similarly, convenience samples (such as asking friends for information on Pokemon-go data) will not give a representative sample of our population.

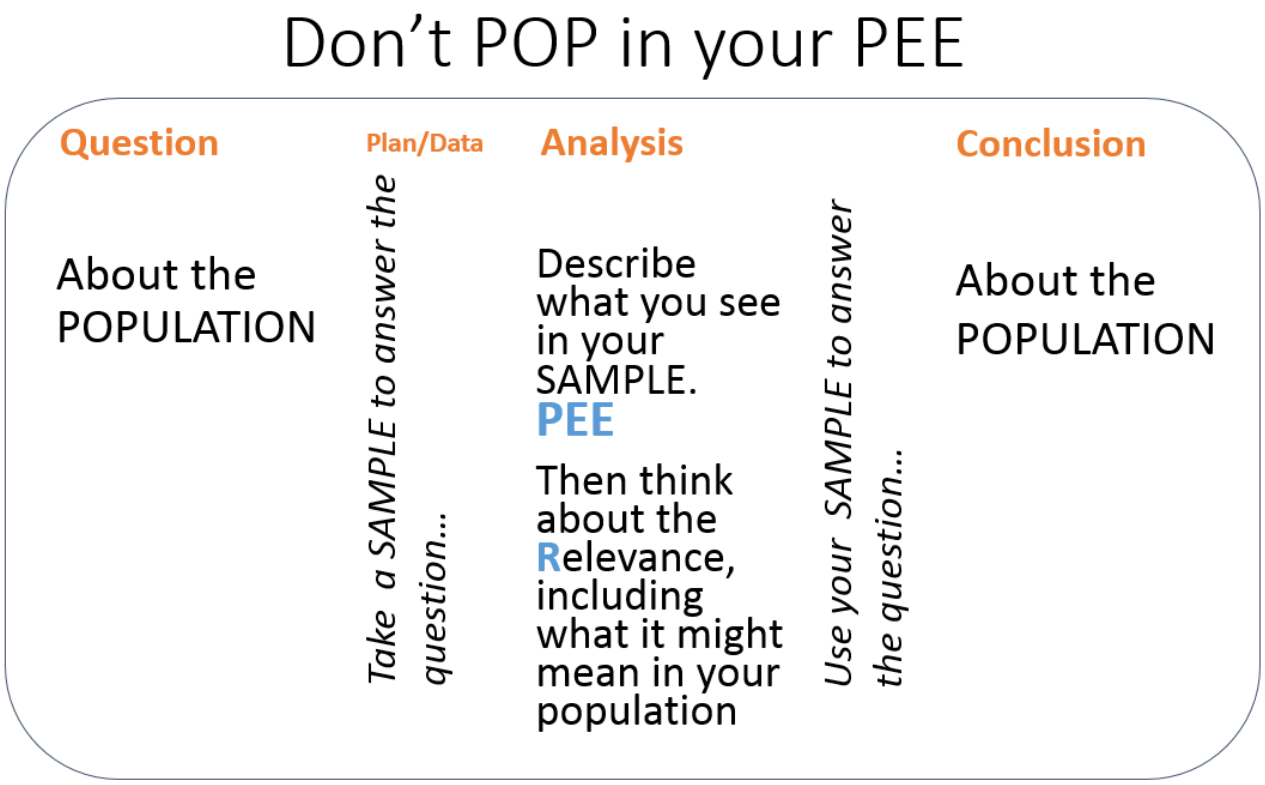
Reinforcing & developing ANALYSIS statements

- Comparative descriptions of sample distributions
  → always use variable, value, unit
  → centres (medians), shift/overlap (position of middle 50% relative to each other), spread (IQR – consistency of sample), shape, unusual
  Descriptions should start from a visual inspection of data displays (dot plots, box plots) with the summary statistics used to support and clarify their observations.
- Writing prompts are useful to support student descriptions
  → PEER = Point, Explain, Evidence + Relevance (so what? Why?)
  → Teacher modelling full sentences – PEE is often incorporated into our first sentence.
  "*From my sample…these boys' heights*

Watch out for…

- Direction in the question not matching up with the direction of difference seen in the samples
  → continually encourage students to connect between their data displays and their written comments

- IQR and middle 50% mix up – IQR is a measure of how consistent/inconsistent the sample is (how long is the box?); middle 50% gives the location of the middle half of the sample (where is the box?).
- Students struggle to describe shape of their sample, especially when dealing with small samples. Descriptions should include key terms such as symmetry and positive or negative skew, and use numbers to support.
  → Be clear when you are moving from describing the shape of your sample to inferential thinking in regard to what shape you might expect to see in the population distribution.
- Students thinking that if they "can't make the call" then they should take another sample → reinforce throughout the development of new ideas that we are working in "teaching world" with a clearly defined known population so that we can develop our new ideas and that they have confidence in their methods when they apply them in the "real world" (a.k.a. "assessment world" where students pretend they are in the "real world")



Curriculum Level 6 (Year 11)

New ideas

- Different size samples give different results → the larger the sample size, the smaller the sampling variation between samples
- Different variation within samples (ie larger IQRs) give different results → the larger the variation within a sample, the larger the sampling variation between samples
- Two ideas above combine to introduce the **Level 6 guide** to "making the call" (based on position of medians, spread and sample size)
  These two ideas are very important concepts to begin developing with students. Students should be introduced to these through class investigation, exploring what happens with multiple samples under different situations.

Reinforcing & developing ANALYSIS statements

- Comparative descriptions of distributions
  → always use variable, value, unit
  → centres (medians), shift/overlap (position of middle 50% relative to each other), spread (IQR – consistency of sample), shape, unusual
  Descriptions should start from a visual inspection of data displays (dot plots, box plots) with the summary statistics used to support and clarify their observations.
- Writing prompts are useful to support student descriptions
  → PEER = Point, Explain, Evidence + Relevance (so what?)
  → Teacher modelling full sentences – PEE is often incorporated into our first sentence. "*From my sample…*"
- Context should be chosen so that students can be ordinarily have enough general knowledge to be able to relate their observed samples to the context (eg: is this difference in medians relatively big/small? Is this what you would expect? Because...)

Watch out for…

- IQR and middle 50% mix up – IQR is a measure of how consistent/inconsistent the sample is (how long is the box?); middle 50% gives the location of the middle half of the sample (where is the box?).
- Students struggle to describe shape of their sample, especially when dealing with small samples. Descriptions should include key terms such as symmetry and positive or negative skew, and use numbers to support.
  → Be clear when you are moving from describing the shape of your sample to inferential thinking in regard to what shape you might expect to see in the population distribution.
  → Discuss this with students when modelling with larger samples and encourage students to think about signal vs noise – what distributional shapes are they seeing that they would expect to see back in the population? – what distributional shapes are likely to just be a result of the sampling process?
- Don't "POP in your PEE" – student ANALYSIS descriptions (PEE) often include the population (*from my sample I notice that the median of the height of boys in NZ is 10 cm taller than the median of the height of girls in NZ)*
  → explain to students that they are developing an understanding of using their sample to tell them things about the population, they need to keep clear in their mind and their descriptions when they're talking about this sample versus back in the population. By keeping the "POP" out of your "PEE" helps keep this distinction. Bring the "POP" back in the Relevance sentence.
- Students' misconception around the relationship between variability within a sample (spread), variability between samples (sample variation) and sample size. It is incorrect for students to conclude that if they could take a larger sample the spread (IQR) will be smaller.
- Ideally, if students know and understand the Level 6 guide for making the call, this is the one they should use BUT they often make mistakes with it. I encourage students to start with the Level 5 guide and then move to the Level 6 guide. If these give a conflicting conclusion – they need to understand that the most sophisticated guide (ie Level 6 guide) is the one that they rely on.

Curriculum Level 7 (Year 12)

See further resources:
http://seniorsecondary.tki.org.nz/Mathematics-and-statistics/Achievement-objectives/AOs-by-level/AO-S7-1

New ideas

- Details of a selection of the main types of sampling methods and what conditions we look for to have a representative sample; any sample-to-population inference we make is only as good as our sample we have.
- The need for an interval of plausible/believable values for our population parameter
- Development of an informal confidence interval (ICI) for the population median which incorporates spread (IQR) and sample size (see points below)
  Students develop the ideas of a ICI that shrinks with increased sample size (as the sampling variation reduces) and increases with increased spread (as the sampling variation increases) through class investigation.
- Testing ICI with a known population
  → ICI captures the population median most of the time
- Interpreting the ICI
  → making a sample-to-population inference ("*We're pretty sure that the population median ring finger length for all Year 12 boys in NZ is somewhere between 9.3cm and 12.4cm*")
- Comparative investigative question needs to be about the difference in population parameters and clearly include variable, groups, population, parameter (median), direction ("*Is the median ring finger length for Year 12 boys in New Zealand higher than the median ring finger length for Year 12 girls in New Zealand?*")
- When comparing two samples from A and B, we look at the ICI for the median of A and B → can say that the median of A is likely to be bigger (smaller) than the median of B back in the population if there is no overlap in ICIs.

Reinforcing & developing

- Different size samples give different results → the larger the sample size, the smaller the sampling variation between samples
- Different variation within samples (ie larger IQRs) give different results → the larger the variation within a sample, the larger the sampling variation between samples
- Context should be chosen so that students can be ordinarily have enough general knowledge to be able to relate their observed samples to the context (eg: is this difference in medians relatively big/small? Is this what you would expect? Because...)

Watch out for…

- Don't "POP in your PEE"
- Encourage students to think about signal vs noise – what distributional shapes are they seeing in their sample that they would expect to see back in the population? – what distributional shapes are likely to just be a result of the sampling process?
- Students misconception around the relationship between variability within a sample (spread), variability between samples (sample variation) and sample size. It is incorrect for students to conclude that if they could take a larger sample the spread (IQR) will be smaller.
- Causation arguments creeping in, under the guise of "sense making" – need to remind students that these are what have been observed and there are other variables in the mix.

<u>Curriculum Level 8 (Year 13)</u>

New ideas

- Development of a method of constructing a formal confidence interval based on the resampling distribution of the statistics of interest – bootstrap confidence interval
- The resampling distribution mimics the repeated sampling distribution, so we can use this as our best estimate of sampling variation.

- Constructing a formal confidence interval for the population parameter of interest: the DIFFERENCE in means or medians
  → making a sample-to-population inference for this parameter ("*We're pretty sure that the mean height of all Year 12 boys in NZ is somewhere between 9.3cm and 12.4cm more than the mean height of all Year 12 girls in NZ*")
- Comparative investigative question needs to be about the difference in population parameters and clearly include variable, groups, population, parameter (median), direction ("*What is the difference between the mean height of Year 12 boys in NZ and the mean height of Year 12 girls in NZ?*").
  It can help to reinforce verbally *"What is the* **size** *of the difference….."*
- When comparing two samples from A and B, we look at the formal confidence interval for the difference in means/medians of A and B
  → can say that the mean/median of A is bigger (smaller) than the mean/median of B back in the population if the formal confidence interval is completely positive (or completely negative).
- Researching context
  → relating the context-research-findings to what is seen in the data and to any conclusions made

Reinforcing & developing

- The need for an interval of plausible/believable values for our population parameter
- Context should be accessible to students so they can use research to explain the features of the observed samples
- The need for a representative sample
- The third related variable idea, to dispel notions of causation, good to get students to have the same variable (eg height) but compare by two different grouping variables (eg by male & female or by Yr9 & Yr 13)
- The size of the difference in real terms – starting to consider practical usefulness/importance

Watch out for…

- Don't "POP in your PEE"
- Students stating "There is no difference" when 0 is in their confidence interval – remind them the confidence interval gives a range of plausible/believable values, 0 is just one of many of these values.
- Avoid teacher modelling the "0 is in or out" justification for making the call (students have not seen this unless we show them) and focus on the direction/sign of the values within the confidence interval.
- Student yes/no questions "*Is there a difference between….*"
- Students using lower level rules to make the call. These are not appropriate e.g. constructing individual intervals and comparing overlap, using level 6 guide for difference of median compared to spread etc. – the focus here is on estimating the size of the difference